Math Success

by DMTI

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Ratio Tables: Building Multiplicative Reasoning that Transfers to Slope

PROBLEM	SOLUTION	PAY OFF
Multiplication Division Proportion Slope – Algebra whole #s, decimals, fractions, rates, percentages	Ratio Tables One structure that bridges arithmetic to algebra	Multiplicative Thinking Grades 3 to Algebra Students reason flexibly—not just recall facts

How Do You Build Multiplicative Thinking?

This builds on facts students know… and builds new facts (a fluency)

1st →

Iconic Model

→ Bar Model

Students see iterations of a unit visually before any symbols

→

2nd →

Symbolic Model

→ Ratio Table

Students encode the same relationship numerically

Example: 7 × 4 = ?

Bar Model (Iconic):

4	4	4	4	4	4	4

4×4 = 16 **3×4 = 12**

16 + 12 = 28 ✓

Ratio Table (Symbolic):

	1	2	3	4	5	7
×4	4	8	12	16	20	28

Compose: 4+3=7 → 16+12=28

Why Is This So Effective?

1 Dual Coding: Iconic → Symbolic

Students first see the relationship (bar model), then encode it numerically (ratio table). Two representations, one coherent idea—this is what makes understanding stick.

2 RME — Realistic Mathematics Education

Freudenthal, Gravemeijer, Treffers: students construct meaning through guided reinvention. The ratio table emerges naturally from students' own reasoning—it is not imposed as a procedure.

3 Builds Reasoning Across All Operations

The same structure supports multiplication, division, proportion, and slope. Nothing needs to be unlearned as students advance from Grade 3 to Algebra.

What Is a Ratio Table and Why It Matters

A ratio table builds multiplication skills and flexibility by giving students a structured way to figure out products using relationships they already know. If a student knows that $2 \times 3 = 6$ and $5 \times 3 = 15$, they can compose these relationships to find $7 \times 3 = 21$ (because $2 + 5 = 7$ and $6 + 15 = 21$). That is multiplicative thinking—composing units to build new relationships—and ratio tables make it visible, teachable, and flexible.

Key points:

- ✓ **Multiplicative thinking, not memorization.** Students use relationships they already understand to build new ones. They are not recalling isolated facts—they are reasoning from known relationships to unknown ones.
- ✓ **A flexible algorithm, not a fixed procedure.** There is no single path through a ratio table. Students decide how to build, combine, or scale values—and that flexibility is what develops strong number sense.
- ✓ **One structure, every number type.** The same ratio table used for whole-number multiplication in Grade 3 extends naturally to decimals in Grade 5 and ratios in Grade 6. The structure stays consistent—nothing needs to be unlearned.
- ✓ **Student thinking made visible.** Ratio tables reveal how students are reasoning. You can see the relationships they understand, where they are making connections, and where they need support—making instruction more precise and responsive.

The 1–2–5–10 Table: Building New Combinations

Build four landmark columns, then compose any product from those anchors.

×8 — The Landmark Table:

1000 × 8 (scaling by powers of 10):

	1	2	5		10		1	10	100	1000
×8	8	16	40		80×8		8	80	800	8,000

now combinations →

0.01 × 8 (decimals):

	1	0.1	0.01
×8	8	0.8	0.08

12 × 8

	2	10	12
×8	16	80	96

2 + 10 = 12 → 16 + 80 = 96

Key insight: The same landmark structure—×1, ×2, ×5, ×10—works for whole numbers, decimals, and fractions. One structure, every number type. Nothing needs to be unlearned.

Proportion and Slope: From Ratio Table to y = mx

Context: Orange juice mixture — 2 grapefruits to 5 oranges

Build the ratio table:

	1	2	5	10
Gr.	1	2	5	10
Or.	2.5	5	12.5	25

÷2 → ×5 →

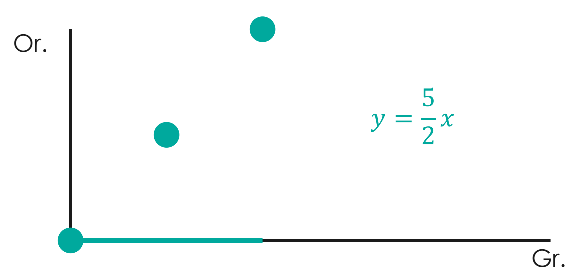
If 100 grapefruits →

	1	10	100
Gr.	1	10	100
Or.	2.5	25	250

If 55 oranges →

	2	10	20	22
Gr.	2	10	20	22
Or.	5	25	50	55

The constant ratio in every table column **is the slope**.
Ratio tables → proportional reasoning → y = mx
m = $\frac{y}{x} = \frac{5}{2} = 2.5$
Graph, $y = \frac{5}{2}x$



The Developmental Progression: From Facts to Slope

Grade 3	Grade 4–5	Grade 5–6	Grade 6–7	Grade 7–8
Multiplication (equal groups)	Multi-digit & Decimals	Fractions & Rates	Proportion & Ratios	Slope & y = mx

One structure — the ratio table — carries students through every level of this progression.

Grade 3–4: 7 × 4

	1	2	4	5	7
×4	4	8	16	20	28

5+2=7 → 20+8=28

Grade 6–7: Proportion

	1	2	5	10
Gr.	1	2	5	10
Or.	2.5	5	12.5	25

Constant ratio = 2.5 per grapefruit

Grade 7–8: Slope

$$y = \frac{5}{2}x$$

$m = \frac{5}{2}$

(the unit rate from the ratio table)

Teacher takeaway: When students see the same structure from multiplication in Grade 3 reappear as slope in Grade 7, mathematics stops feeling like a collection of unrelated procedures. Ratio tables make that connection visible.

Teacher Questions

Use these prompts to draw out student reasoning during ratio table work

GETTING STARTED	MAKING THINKING VISIBLE	PUSHING FLEXIBLE THINKING	CONNECTING STRUCTURE
"What does 1 represent in this table?"	*"How are the two columns related? Describe the multiplicative relationship."*	*"Which columns did you use? How could you get the same answer using fewer columns?"*	*"What do you notice about the products as the multiplier gets ___ times larger?"*
"Which columns feel like good starting points? Why?"	*"Point to where in your table you found that."*	*"How can you find another way to compose the same product using different columns?"*	*"If you multiply by 100, how many place values do you increase?"*
"How did you use the 1–2–5–10 table to build this?"	*"I see you wrote ___. Walk me through how you got there."*	*"If you doubled this factor, what would happen to the product?"*	*"What stays the same in every column? What is that called in algebra?"*

Standards Addressed

Grade 3

3.OA.C.7 Develop fluency with multiplication facts within 100.

3.OA.D.9 Identify and explain arithmetic patterns in multiplication tables using properties of operations.

Grade 4

4.OA.C.5 Generate and analyze multiplicative number patterns.

4.NBT.B.5 Multiply multi-digit whole numbers using place value and properties of operations.

Grade 5

5.NBT.B.5 Fluently multiply multi-digit whole numbers using efficient strategies.

5.NBT.B.6–7 Apply strategies to decimals and fractions using ratio table structure.

Grade 6–7

6.RP.A.3 Use ratio tables to represent equivalent ratios and reason multiplicatively.

7.RP.A.2 Recognize and represent proportional relationships; connect to y = mx.

Seven-deck series spanning Grades 3–6 | www.dmtinstitute.com | Teachers Pay Teachers

A → B → C → D → E → F → G

A Doubling

Building the first multiplicative structure

Students build ratio tables by doubling — the simplest and most powerful multiplicative relationship. This is where students first experience multiplication as scaling, not repeated addition.

Grades 3–6 | 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 · 5.NBT.B.5 · 6.RP.A.3

B Multiples in Order

Fluency through sequential patterns

Students list all multiples in order and look for patterns in the digits, odd/even products, and how the two rows relate. Builds the pattern recognition that enables flexible use of the table later.

Grades 3–6 | 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 · 5.NBT.B.5 · 6.RP.A.3

C Composing Multiples

From listing to problem-solving

Students use a completed sequential table to solve larger problems by combining columns. The critical bridge from filling in a table to actually using one as a problem-solving tool.

Grades 3–6 | 3.OA.C.7 · 3.OA.D.9 · 4.NBT.B.5 · 5.NBT.B.5 · 6.RP.A.3

D 1–2–5–10 Table

The full flexible algorithm

Students build four landmark columns (×1, ×2, ×5, ×10) and compose any product from those anchors. This is the full flexible algorithm for whole-number multiplication.

Grades 4–6 | 4.OA.C.5 · 4.NBT.B.5 · 5.NBT.B.5 · 5.NBT.B.6 · 5.NBT.B.7 · 6.RP.A.3

E Powers of 10

Scaling whole numbers

Students apply the ratio table structure to powers of 10 with whole-number bases — scaling by 10, 100, 1,000, and more. Same structure as Deck D; only the multipliers change.

F Powers of 10

Scaling decimals

Students apply the ratio table structure to decimal bases, multiplying by powers of 10. The same reasoning used for whole numbers in Deck E now extends to decimals — no new procedure needed.

G Division

Division is scaling down

Students use the ratio table to divide by powers of 10. Rather than treating division as a separate operation, this deck frames it as scaling down — the same structure from Decks E and F, running in reverse.