Jonathan Brendefur, PhD, and Colleagues
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## Why This Matters

When a student writes 1→4, 2→6, 3→8, and sees nothing wrong, they are not confused — they are following the logic of instruction that never made the multiplicative relationship explicit. Most ratio-table teaching focuses on filling in cells rather than on reasoning about units, relationships, and structure. The result: students who can complete a table but cannot use one.

**Core Claim:** Durable ratio table learning begins when students understand that every column must match a single unit relationship — reason from the unit, not count up to it.

*Meaning → Structure → Flexibility → Efficiency*

## The Unit Relationship

**Start with the unit, not the procedure.** Every ratio table is built on one question: what does 1 represent? Every other column must be verified against that unit — not generated by adding a constant. Flexibility only develops once the unit anchor is firmly established.

**Key insight:** When students see the unit column as the anchor, strategy becomes a choice. Ask: "What does 1 represent? How does every column connect back to it?"

## The Instructional Sequence: Five Phases

Use this sequence consistently across Grades 3–6. Each phase builds on the last.

### Phase 1: Build Meaning with Visual Models

**Goal:** Establish multiplicative relationships iconically before introducing the table. Use bar models, and double number lines — students visualize that all units (groups) are identical before any symbolic encoding.

*Sample task:* 1 bag has 3 apples. Draw 4 bags using a bar model. Students move from drawing → repeated addition → early multiplication, always seeing the unit before encoding it.

**Key questions:** *"What is one unit (group) worth?" · "How many units (groups)?" · "How are all the units (groups) the same?"*

## Phase 2: Introduce the Ratio Table

**Goal:** Transition from visual models to symbolic structure. Introduce the unit column first: "This column tells us the unit. Every other column has to match it." Start with 1–4, letting students count up — this establishes the unit anchor before introducing flexibility.

| Bags | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| Apples | 3 | 6 | 9 | 12 |

**The 1–4 building phase is essential.** It grounds every subsequent jump in a structure the student can verify.

## Phase 3: Move Beyond Sequential Counting

**Goal:** Shift from iteration to flexible, strategic reasoning — the transition from sequential counting to multiplicative reasoning.

**Once students have a 1–4 table, prompt jumps:** *"Do we have to go 1, 2, 3, 4? What if we need 10? What if we need 7 — could you decompose it into parts you already know?"*

| Bags | 1 | 5 | 10 | 7 |
|---|---|---|---|---|
| Apples | 3 | 15 | 30 | ? |

Students discover strategies like 7 = 5 + 2 or 7 = 10 − 3, using columns already built. This is where the ratio table becomes a **flexible algorithm** — there is no single correct path.

## Phase 4: Composition and Decomposition

**Goal:** Develop efficiency through deliberate number composition. Students choose which parts to build, not follow a prescribed path.

*Task:* If 1 → 4, find 13 × 4. Compose: 10 → 40, then 3 → 12, combine for 52.

| ×4 | 1 | 10 | 3 | 13 |
|---|---|---|---|---|
| | 4 | 40 | 12 | 52 |

**Teacher prompts:** *"How did you decompose 13?"* · *"Which numbers are easier?"* · *"How could someone solve it differently and get the same answer?"*

## Phase 5: Scaling and Connecting to Rate

**Goal:** Work in both directions and connect to unit rate, proportion, and linear functions.

*Task:* If 4 → 20, what is 1 → ? What is 8 → ? Students divide to find the unit, then scale up. Have them plot table pairs on a graph — the straight line makes the slope connection visible.

| | 1 | 4 | 8 |
|---|---|---|---|
| | 5 | 20 | 40 |

**Teacher prompts:** *"How did you get back to 1?" · "What stays the same in every column?" · "What would you see if you graphed these pairs?"*

## Teacher Moves That Build Ratio Table Reasoning

These moves should be present in every lesson — not just when introducing ratio tables for the first time.

| | |
|---|---|
| **Always anchor to the unit** | Every lesson returns to: "What does 1 represent?" If a student's table breaks down, return to the unit column and rebuild from there. |
| **Press for structure** | After answers, ask how students built or scaled the table. Make structure visible: "How did you scale that?" · "What did you compose?" · "Where did that column come from?" |
| **Encourage multiple pathways** | Accept different strategies, then show two or three side-by-side. When students see that different paths reach the same answer, they develop trust in the structure. |
| **Delay formal procedures** | Resist moving to cross multiplication before students have built a strong relational understanding. Ratio tables provide a meaningful path that doesn't require memorized rules. |

## Common Misconceptions and How to Address Them

| Misconception | What You See | Response |
|---|---|---|
| **Additive thinking** | 1→4, 2→6, 3→8 (adding 2 instead of ×4) | Return to the unit: "Does 2 × 4 = 6? Let's check the first column." Compare two columns explicitly against the unit. |
| **Missing unit column** | Student builds from a convenient column, skips starting at 1 | Make 1 non-negotiable: "We always need to know what 1 is worth. Let's start there." |
| **Sequential counting only** | Counts 1, 2, 3… all the way to 37 to find 37 × 6 | Prompt jumps: "That would take a long time. What if we jumped to 10? To 30? What would you still need?" |
| **Compounding errors** | Multiple columns wrong because an early error carried through | Cover all columns except the unit. Rebuild one at a time, checking each against the unit before moving on. |

## A Sample Three-Day Lesson Sequence

Each day follows a Launch → Explore → Discuss → Close structure. Works for Grades 3–6.

| | |
|---|---|
| **Day 1**<br>From Visual to Table | **Goal: Build a ratio table for a simple multiplicative context; explain the unit.**<br>• Launch (5 min): Equal-units (groups) context — students draw the visual first.<br>• Explore (15 min): Introduce the table. Students record 1–4 columns, connecting each to their bar model.<br>• Discuss (10 min): "What is the unit? How does every column connect back to it?"<br>• Close (5 min): Students write one sentence explaining what a single column means in context. |
| **Day 2**<br>Building Flexibility | **Goal: Use jumps (×2, ×5, ×10) and decomposition to extend a table efficiently.**<br>• Launch (5 min): Review a Day 1 table. "What if I need 10? How can I find 10 without counting up?"<br>• Explore (15 min): Students extend tables using jumps; compose non-landmark values (7, 13, 23).<br>• Discuss (10 min): Share two or three strategies for the same target. "Did anyone decompose it differently?"<br>• Close (5 min): Exit task — solve one missing-value problem using a strategy other than counting up. |
| **Day 3**<br>Generalization & Connection | **Goal: Apply flexible reasoning to a new context; begin connecting to rate.**<br>• Launch (5 min): New context (cost/item, distance/hour) — identify the unit relationship first.<br>• Explore (15 min): Students use the flexible algorithm with multiple compositions.<br>• Discuss (10 min): "What stays the same in every column? What would happen if you graphed these pairs?"<br>• Close (5 min): Describe the relationship in their own words — both the unit and the pattern. |

## What Success Looks Like

A student who has internalized ratio table reasoning will consistently:

- Begin by identifying the unit (the "1 column") before extending the table
- Move beyond sequential counting — use efficient jumps and compositions
- Explain why their answer is correct by referring to the unit relationship
- Maintain equivalence across every column without checking every step
- Apply the same structure across contexts: equal units, price, distance, rate

When you see students doing these things, the ratio table stops being a task type and becomes a **thinking tool**.

## Practice Resources from DMTI

Seven practice decks develop ratio-table fluency across Grades 3–6, each building on the previous one. Available at **www.dmtinstitute.com** and **www.teacherspayteachers.com** (search DMTI, Ratio Tables).

| | Title | What It Builds | Standards |
|---|---|---|---|
| A | Doubling | First multiplicative structure. Entry point for the whole series. | Gr. 3–6 \| 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 |
| B | Multiples in Order | Sequential multiples, digit patterns, odd/even products. Builds fluency before flexible use. | Gr. 3–6 \| 3.OA.B.5 · 3.OA.C.7 · 3.OA.D.9 · 4.OA.C.5 · 4.NBT.B.5 |
| C | Composing Multiples | Use a completed table to solve larger problems by combining columns — from filling to problem-solving. | Gr. 3–6 \| 3.OA.C.7 · 3.OA.D.9 · 4.NBT.B.5 · 5.NBT.B.5 · 6.RP.A.3 |
| D | 1–2–5–10 Table | Build four landmark columns; compose any product from those anchors. The full flexible algorithm. | Gr. 4–6 \| 4.OA.C.5 · 4.NBT.B.5 · 5.NBT.B.5 · 5.NBT.B.6–7 · 6.RP.A.3 |
| E | Powers of 10 (Whole #s) | Apply the ratio table to powers of 10 with whole-number bases — scaling by 10, 100, 1,000+. | Gr. 4–6 \| 4.NBT.B.5 · 5.NBT.B.2 · 5.NBT.B.5 · 5.NBT.B.6 · 6.RP.A.3 |
| F | Powers of 10 (Decimals) | Same multiplicative reasoning from Deck E, extended to decimal bases. No new procedure needed. | Gr. 5–6 \| 5.NBT.B.2 · 5.NBT.B.5 · 5.NBT.B.6–7 · 6.RP.A.3 |
| G | Division | Division as scaling down — the same structure from Decks E–F running in reverse. | Gr. 5–6 \| 5.NBT.B.2 · 5.NBT.B.6–7 · 6.NS.B.2 · 6.RP.A.3 |

**Decks A–B** build the Grade 3 foundation. **Decks C–D** develop the full flexible algorithm. **Decks E–F** extend to powers of 10. **Deck G** frames division as scaling down. Each deck uses the same structure — nothing needs to be unlearned as students advance.

***"What is the unit? What do I know? How can I build what I need?"***

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