



Developing Mathematical Thinking:
When Math Sticks: The Cognitive Science of Durable
Learning

RESEARCH OVERVIEW

BY

Jonathan Brendefur, PhD
and Associates

WHEN MATH STICKS: THE COGNITIVE SCIENCE OF DURABLE LEARNING

(RESEARCH OVERVIEW)

Introduction

Effective math learning is not simply the absorption of procedures—it is the process of encoding, organizing, and flexibly retrieving mathematical knowledge through the interplay of cognitive mechanisms. **Drawing on cognitive science, learning begins within working memory's constraints and, with purposeful instruction, develops into durable, adaptable long-term memory.** This overview brings together foundational principles from theorists such as Jerome Bruner, David Ausubel, Jean Piaget, and John Sweller, illustrating how schema theory, assimilation, accommodation, cognitive load, and retrieval practice underpin powerful mathematics instruction for lasting understanding and flexible application (Bruner, 1966; Ausubel, 1968; Piaget, 1965; Sweller, 1988).

What Is the Psychological Foundation for Learning Math?

How do students' mental structures grow and adapt as they encounter new mathematical ideas, and what does this mean for effective teaching?

Learning mathematics is fundamentally a process of building and reorganizing mental representations—schemas—through experience and reflection (Ausubel, 1968; Piaget, 1965). According to schema theory, every learner enters the classroom with existing networks of prior knowledge, which act as frameworks for understanding and integrating new ideas (Ausubel, 1968; Baddeley & Hitch, 1974). Piaget's concepts of assimilation and accommodation provide further insight: assimilation occurs when new information fits within a learner's existing schema, strengthening and expanding their network effortlessly. When a concept resists easy categorization—say, when a student encounters non-standard models of multiplication—accommodation is required; the schema itself must be revised or expanded to integrate this fresh insight (Piaget, 1965).

Instruction that leverages these natural processes scaffolds learning with intention. Bruner's theory guides teachers in sequencing learning from **enactive** (action-based, physical manipulation) to **iconic** (visual representation) to **symbolic** (abstract formalism). At every stage, teachers can provide opportunities for both assimilation—by linking new content to familiar procedures—and accommodation—by challenging students to resolve contradictions between their prior conceptions and novel ways of thinking. Ausubel's meaningful learning involves explicit efforts to connect, reorganize, and strengthen schemas through advance organizers and discourse that surface and address misconceptions, making mathematics more logical and memorable (Ausubel, 1968; Bruner, 1966; Piaget, 1965).

How Does a Math Concept Move from First Encounter to Mastery?

How do math ideas transition from fleeting exposure in working memory to robust, operating knowledge, and how do assimilation and accommodation play a role in this journey?

Early mastery begins in working memory, where a student can hold and manipulate only a few pieces of information at a time (Baddeley & Hitch, 1974; Cowan, 2001). Effective instruction reduces cognitive load by using enactive representations—manipulatives and hands-on activities that allow students to “offload” thinking onto physical objects (Bruner, 1966; Sweller, 1988). As learners’ skills deepen, **iconic models such as diagrams and area models help them assimilate mathematical relationships by “chunking” details into unified, existing schemas** (Piaget, 1965).

However, true mastery demands more than assimilation alone. When a learner encounters problems that cannot be resolved by their current schemas—such as an unfamiliar algorithm or a counterintuitive approach—**accommodation** becomes necessary. In these moments, **students must modify existing schemas or create new ones, thereby advancing their understanding and preparing for higher-level abstraction** (Piaget, 1965). Teachers nurture this process by fostering reflection, encouraging explanation, and sequencing challenges that require rethinking and reorganization. Over time, these adaptive changes result in a more flexible and integrated network of mathematical knowledge, ready for transfer and creative problem-solving (Ausubel, 1968; Bruner, 1966; Roediger & Karpicke, 2006).

Why Is a Psychological Approach Effective?

Why are instructional methods that prioritize schema growth, assimilation, and accommodation proven to yield lasting mathematical mastery?

When teaching supports both assimilation (connecting new ideas to known structures) and accommodation (adapting or restructuring those structures), students develop deep, interconnected schemas. **Mental networks enable students to access mathematical knowledge through multiple pathways**—retrieving a fact via familiar visuals, an analogy, or prior experience. This versatility translates directly to flexible problem-solving and adaptability in new or novel contexts (Bruner, 1966; Ausubel, 1968).

Supporting these mechanisms also reduces cognitive overload and fosters positive dispositions toward mathematics. By grounding abstract mathematical concepts in concrete experiences and familiar schemas, teachers enable confident assimilation and minimize confusion (Sweller, 1988; Piaget, 1965). When new information requires accommodation, **targeted guidance and reflective learning experiences ensure that schema expansion happens deliberately, not randomly**. Cognitive science further emphasizes the importance of retrieval practice, spaced repetition, and interleaved practice—techniques that have been shown to solidify schema changes, reinforce long-term retention, and maintain active learning (Roediger & Karpicke, 2006).

What Are the Disadvantages and Challenges?

What obstacles must educators face when seeking to foster assimilation, accommodation, and schema growth in mathematics classrooms?

The shift to intentional, **cognitively based instruction requires rich lesson design, time for scaffolded experiences, and opportunities for both assimilation and accommodation to occur**. Still, the pressure to cover content rapidly or prepare for tests may undermine the slow, recursive nature of schema transformation (Sweller, 1988; Rohrer et al., 2015). Traditional assessments often prioritize memorized procedures, leaving little incentive for students to reflect or restructure their schemas when they encounter contradictory approaches (Bruner, 1966; Piaget, 1965).

Another hurdle is the need for sustained pedagogical knowledge and professional development. Teachers must be able to identify when a student is simply assimilating (adding to current knowledge) and when accommodation (schema change) is required, offering targeted supports for each. Without system-level curriculum alignment and formative assessments that reward deep conceptual transformations, the powerful benefits of schema theory and adaptive learning can remain underutilized. Cultivating a classroom and school culture that values not just “right answers,” but the process of cognitive growth—including errors, reflection, and schema revision—is vital to unlocking genuine mathematical mastery (Ausubel, 1968; Bruner, 1966; Sweller, 1988).

What Are the Implications for Curriculum and Teaching Practice?

How can curriculum and classroom practice be designed to foster both assimilation and accommodation, building robust schemas for mathematics learning?

Instruction should sequence learning from concrete to visual to abstract, giving students repeated opportunities to assimilate familiar ideas and then challenging them to accommodate new, unfamiliar ones. This means **incorporating advance organizers, conceptual scaffolding, productive struggle, and explicit discourse at every stage**—tools that surface students’ thinking and make schema growth visible (Bruner, 1966; Ausubel, 1968; Piaget, 1965).

Assessment should recognize and foster both mechanisms—valuing not only rapid retrieval, but the ability to revise, connect, and reorganize knowledge when needed. **Frequent, formative checks for understanding, ongoing practice with multiple representations, and retrieval exercises help solidify schema changes and maintain both assimilation and accommodation.** At the curriculum level, materials and pacing must allow time for recursive and reflective learning, with opportunities for students to both consolidate their existing knowledge and transform their thinking when encountering foundational challenges. Only then can the full power of cognitive psychology be brought to bear—cultivating mathematical thinkers who assimilate, accommodate, and contribute creatively (Bruner, 1966; Piaget, 1965; Ausubel, 1968; Sweller, 1988).

Summary

In summary, the psychology of mathematics learning is grounded in schema theory and driven by the dynamic interplay of assimilation and accommodation. By designing instruction that enables students to both expand and modify existing schemas when confronted with new ideas, educators ensure that mathematical knowledge is flexible, durable, and deeply meaningful. By harnessing the psychology of durable memory—schema building, assimilation, accommodation, and retrieval—educators unlock mathematics as a lasting, empowering discipline for all learners (Bruner, 1966; Piaget, 1965; Ausubel, 1968; Roediger & Karpicke, 2006).

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Are We Teaching Math for the Test or for a Lifetime?

Cramming procedures might boost short-term scores, but cognitive science reveals how to build mathematical understanding that endures. It is not about more practice; it is about better, more brain-aware practice.

Our latest Research Overview breaks down the science of learning into actionable strategies for educators and leaders. Discover:

- **How to leverage working memory's limits**, instead of being defeated by them.
- **Why the sequence of concrete → visual → abstract (Bruner's Modes) is non-negotiable** for deep encoding.
- **How "desirable difficulties" like spaced and interleaved practice** combat the forgetting curve and build fluency that lasts for years.
- **The role of schema theory in creating flexible problem-solvers** who can apply knowledge to novel situations.
- **This is not just theory—it is a blueprint for transforming math instruction.**

What is one strategy you use to help make math "stick" for your students? Share in the comments below!



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