



Developing Mathematical Thinking: The Five Counting Principles

RESEARCH OVERVIEW

BY

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THE FIVE COUNTING PRINCIPLES

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Introduction

The five counting principles proposed by Gelman and Gallistel (1978) form the cornerstone of early numerical understanding. These principles—***one-to-one correspondence, stable order, cardinality, abstraction, and order irrelevance***—are crucial for developing number sense and later mathematical skills. This overview examines the research on these principles, common student difficulties, cognitive psychology insights, and effective teaching strategies.

Student Difficulties and Misconceptions

Research has identified several areas where students struggle with counting principles. One common issue is ***skipping or repeating numbers***, especially with larger sets (Fuson, 1988). For example, a child might count "1, 2, 3, 5, 6" or "1, 2, 3, 3, 4, 5" when presented with a group of objects. Another significant challenge is grasping the concept of cardinality. While many students can recite numbers, they often ***fail to understand that the last number represents the total quantity*** (Bermejo, 1996).

The principle of order irrelevance also poses difficulties for some children. They may ***believe that changing the counting order affects the total***, such as thinking that counting from left to right will yield a different result than counting from right to left (Baroody, 1984). Additionally, the abstraction principle can be challenging, as students sometimes ***struggle to apply counting to diverse or abstract sets of objects*** (Gelman & Gallistel, 1978).

Conservation of number is another area where misconceptions arise. ***Some children think that rearranging objects changes the quantity***, for instance, believing that spreading out a group of counters makes "more" than when they are close together (Piaget, 1952). Difficulties multiply as set sizes increase beyond 5 or 6 items (Fuson, 1988). Furthermore, the transition from counting all to ***counting from a given number is challenging*** for many students (Carpenter & Moser, 1984).

Research by Booker et al. (2004) suggests that reviewing and reinforcing one-digit numbers, including ***zero***, is important when teaching counting. Materials like number lines, bar models, and number words and symbols help learners construct mental images.

Cognitive Psychology Insights and Solutions

Cognitive psychology research offers several insights and solutions to address these challenges. Developing **subitizing** skills—quickly recognizing small quantities without counting—can support overall counting proficiency (Clements, 1999). For instance, recognizing groups of three or four objects at a glance can serve as a foundation for more complex counting tasks.

Strengthening executive function skills, such as working memory and attention regulation, can also improve counting performance (Blair & Razza, 2007). Activities that require students to hold information in mind while performing a task, like counting backward or skip counting, can help develop these skills.

Studies have shown that **backward counting** is more cognitively demanding than forward counting. It requires a stronger grasp of number relationships and mental manipulation of numbers. This increased cognitive load makes backward counting a more sensitive measure of a child's numerical understanding (Clarke et al. 2002).

Language development is crucial in counting abilities, especially for English language learners (Purpura & Reid, 2016). Incorporating rich mathematical vocabulary and providing opportunities for students to explain their thinking can enhance both language and counting skills. Incorporating structural words such as unit, iterating, partitioning, decomposing, composing, and equal (same as), helps reinforce the understanding of counting.

Bruner's (1966) **Enactive-Iconic-Symbolic** approach has shown effectiveness in helping students transition from physical experiences to mental representations and abstract symbols. For example, students might start by counting actual objects (enactive), then progress to counting dots and spaces on the number line (iconic), and finally work with abstract number symbols (symbolic).

Effective Math Models and Language

Initially, enactive models (manipulatives) allow hands-on exploration of counting principles (Carbonneau, Marley, & Selig, 2013). Counting songs and rhymes supports memorizing number sequences and can incorporate finger counting for added reinforcement (Sarama & Clements, 2009). Using clear, consistent language when describing counting actions helps reinforce concepts (Klibanoff et al., 2006). For instance, always saying, "How many are there in total?" after counting reinforces the cardinality principle.

Research suggests several effective models and language approaches for teaching counting. Number lines and number paths help support stable order and one-to-one correspondence (Frye et al., 2013). Ten-frames help students visualize numbers up to five and support subitizing skills (Van de Walle, Karp, & Bay-Williams, 2018). For example, a ten-frame filled with seven counters allows students to quickly see "7" as composing "5 and 2."

However, the number line and bar model allow students to count efficiently, understand number magnitude, and build cardinality. Siegler and Booth (2004) found that children's ability to use number lines helps them understand number magnitude and cardinality. Murata (2004) demonstrates the bar model's effectiveness in building cardinality and number sense.

Counting to everyday situations makes the concepts more relatable and meaningful (Ginsburg & Amit, 2008). Introducing comparison language like "more," "less," "equal," "compose," "decompose," and "iterate" supports the

development of number sense (Mix, Sandhofer, & Baroody, 2005).

Teaching Strategies and Learning Progressions for Counting

Effective teaching strategies based on research findings include explicit instruction, where teachers demonstrate and explain each counting principle (Doabler & Fien, 2013). Guided practice provides opportunities for students to practice with teacher support. Using students' mistakes as teaching opportunities to address misconceptions can be particularly effective (Ashlock, 2010; Brendefur & Strother, 2022).

Starting with small sets and gradually increasing difficulty helps build confidence and skills (Clements & Sarama, 2014). Using different types of objects and arrangements reinforces abstraction and order irrelevance (Mix, 2009). Asking "how many?" after counting reinforces cardinality (Bermejo, Morales, & Garcia de Osuna, 2004). Encouraging students to explain their counting strategies to each other promotes deeper understanding and peer learning (Webb, 1991).

McAlpine (2010) recommends focusing on questions for early numeracy learners: "What is the number before and after a particular number? What does each digit represent in a number? These questions align well with developing counting on and counting back skills.

Learning Progressions for Counting

Research on the learning progression for counting reveals a structured developmental sequence that forms the foundation for more advanced mathematical concepts. This progression typically unfolds as early counting skills, advanced counting skills, and transition to place value.

Early counting starts with pre-counting. Children develop an understanding of concepts like "more," "less," and "the same" through comparison without actual counting. Then, transition to one-to-one Counting, where children learn to assign one counting word to each object in a set. Next, work on stable order where children consistently recite the counting sequence (forward and backward), often mastering numbers 0-10 between ages 3 and 4. And, finally, work on cardinality. Have children understand that the last number counted represents the total quantity in a set. Using a bar model to represent quantities works best.

While building **advanced counting skills**, children should focus on counting on, where they learn to count forward from a given number rather than always starting from one. While counting forward, then have children count backward. This skill emerges later and is considered a better indicator of numerical understanding. Soon after, have children practice skip counting by 2s, 5s, and 10s forward and counting. Again, the bar model is one of the most useful models for building these concepts.

The counting progression leads to an understanding of **place value**. In kindergarten, children learn to see teen numbers as units of ten and units of one. In first grade, children understand that ten is a unit, and each digit in the tens place represents ten. In second grade, the place value concept extends to hundreds, with children recognizing that 10 units of ten compose a new unit of one hundred.

Finally, developing counting skills and understanding place value directly supports **addition and subtraction**. Initially, children solve addition problems by counting all objects in both sets. As skills progress, children learn to start with a

larger number and count on it. By third grade, children use their understanding of place value to add and subtract three-digit numbers fluently.

Conclusion

The five counting principles form a critical foundation for mathematical understanding. By addressing common misconceptions, incorporating cognitive psychology insights, and using effective models and language, educators can support students in developing strong counting skills. This foundation will serve as a springboard for more advanced mathematical concepts and problem-solving abilities (National Research Council, 2009).

Recent research emphasizes the importance of backward counting in developing number sense and preparing students for subtraction concepts. Children learn to understand the inverse relationship between addition and subtraction by counting backward. This skill enhances their mathematical abilities and has practical applications in everyday life, such as time management and countdowns.

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Social Media

What are the **Five Counting Principles**? Moreover, what does the research say about getting children on the right track to building a strong math foundation? Here are a few items that might surprise you: **Counting backward** is a powerful tool for enhancing number sense. The **number line** and **bar model** are the critical math models you should incorporate into your lessons. Dive into this research overview to learn how to address common student difficulties, leverage cognitive psychology insights, and employ effective teaching strategies to transform classrooms.

Join us in exploring these powerful strategies and their impact on mathematical thinking!



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