DMT INSTITUTE NEWSLETTER

## Math Success with DMTI

# SQUARES ARE RECTANGLES? TEACHING QUADRILATERALS FOR REAL UNDERSTANDING

## In today's edition, we explore:

- **The Classroom Reality:** Why students keep classifying quadrilaterals by appearance—and why that always breaks down
- **The Cognitive Why:** How prototype bias and picture-based materials produce misconceptions that resist correction
- **The Instructional Shift:** Teaching shapes as systems of line relationships, not pictures to memorize
- **The Hierarchy Question:** How does making inclusive categories visible change everything students know about squares, rectangles, and rhombi

## Educators

A student confidently labels a rotated square a "diamond."

Another insists a square is not a rectangle because "rectangles are the long ones."

A third can name every shape on the worksheet—but has no idea why a square belongs inside the rectangle or a parallelogram belongs inside a trapezoid category.

These are not careless mistakes. They are logical conclusions drawn from the way quadrilaterals are typically taught.

**When instruction presents shapes as pictures to recognize rather than as structures defined by properties**, students build fragile mental categories that collapse the moment a familiar shape is rotated, stretched, or shown in an unfamiliar form.

Our latest **DMT Insight—Teaching Quadrilaterals for Understanding**—explains why these misconceptions persist and what a more durable instructional path looks like.

## The Classroom Reality: Where the Confusion Begins

In most classrooms, quadrilateral instruction looks like this:

- Posters and flash cards showing one "standard" example of each shape
- Worksheets where every parallelogram leans the same direction
- Shape names treated as mutually exclusive labels

- Squares and rectangles presented as separate, non-overlapping families

From a student's perspective, the message is clear: geometry is about recognizing what shapes look like, not reasoning about what must be true.

So students apply the rules their materials have taught them:

- "A square is a square. It can't also be a rectangle."
- "That tilted shape is a diamond—it doesn't look like a square."
- "I know it's a parallelogram because it looks like the one in the book."
- "A trapezoid always looks like a pattern block."

The materials are consistent—even when the mathematics is not.

## The Cognitive Why: Prototypes Override Properties

Cognitive research helps explain why these errors are so persistent. Two forces work against a durable understanding.

The first is **prototype bias**. When students encounter the same standard image repeatedly—a wide, horizontal rectangle; an upright square; a leftward-leaning parallelogram—they build a mental picture that becomes the decision-making template. Anything that deviates from that template is rejected, regardless of its actual properties.

The second is the gap between **the concept image and the concept definition**. A student may recite "a rectangle has four right angles" perfectly while their mental image still excludes squares and narrow vertical shapes. The definition lives in memory; the prototype governs the decision.

**Variation in orientation, size, and regularity is not enrichment—it is the mechanism by which formal definitions gain genuine meaning.** Without it, no amount of definition-recitation produces durable understanding.

## The Instructional Shift: Teach Lines Before Shapes

Our research overview proposes a foundational reframe: **before shape names, students must understand the line relationships that define them.**

From this perspective:

- **Congruent** means side lengths are the same length
- **Parallelism** means two lines that extend infinitely and never meet, not two sides that look equal in length
- **A right angle** is defined by perpendicular lines—not by whether a corner "looks square"
- **A quadrilateral family** is defined by which line conditions apply—not by which picture it resembles

When instruction consistently asks "What must be true about this shape, no matter how it is oriented?"—students stop sorting by appearance and start reasoning about structure.

During instruction, small questions create large shifts:

- "What lines created this shape?"
- "How do you know those sides are parallel?"
- "What would have to change for this rhombus to become a square?"

**These questions move students from recognizing quadrilaterals to reasoning about them.**

## The Hierarchy Question: Why Are Squares Rectangles

The most persistent conceptual error in quadrilateral instruction is treating category names as mutually exclusive. Students learn that a square is one thing and a rectangle is another—so a square cannot be a rectangle.

When the hierarchy is taught through line relationships, the logic becomes transparent:

- **Rectangle:** a parallelogram whose lines meet at right angles
- **Rhombus:** a parallelogram with all sides equal
- **Square:** satisfies both conditions simultaneously—it is the most constrained member of both families

**Satisfying more constraints makes a shape a stronger member of a category—not a different one.** When students see this logic displayed in nested diagrams—squares inside rectangles, rectangles inside parallelograms—the hierarchy stops being arbitrary and becomes inevitable.

## Your Turn to Share

We would love to hear from you. **What is one quadrilateral misconception you see every year—and what has worked best to address it?**

Share your thoughts at **contact@dmtinstitute.com**. We will compile and elevate your insights to help advocate for stronger teacher support across districts.

**Keep inspiring mathematical minds!**

## MORE INFORMATION

This newsletter is just a preview—there is much more to explore. For the complete **research overview**, **teacher companion guide**, and **classroom poster**, visit the **DMT Insights** page at **dmtinstitute.com/research**.

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