



Developing Mathematical Thinking:
Embracing Mistakes and Misconceptions in
Mathematics: A Catalyst for Building Thinkers

DMTI INSIGHTS

BY

Jonathan Brendefur, PhD
and Associates

EMBRACING MISTAKES AND MISCONCEPTIONS IN MATH EDUCATION: A CATALYST FOR BUILDING THINKERS

(DMTI INSIGHTS)

Introduction

What if being "wrong" is the first step to being right? For decades, mathematics education has centered around correctness—teachers model reliable procedures, students practice for accuracy, and mistakes are quickly corrected. This efficiency, however, comes at a cost: it frames errors as failures, cultivating a classroom climate where intellectual risk-taking is rare and learning can become shallow. Mounting research in mathematics education and cognitive psychology reveals that mistakes and misconceptions are not obstacles; they are essential, catalytic pathways to learning (Borasi, 1996; Boaler, 2016).

In the K–8 grades, where foundational concepts take root, misconceptions are not simply gaps in understanding—they are logical extensions of prior knowledge. By intentionally leveraging and analyzing student errors, educators gain vital insight into students' cognitive processes and create conditions for robust, lasting growth in mathematical proficiency. This overview synthesizes why embracing mistakes is crucial for building conceptual understanding, procedural fluency, and mathematical resilience.

I. Uncovering Student Thinking: Windows into the Mind

What can seemingly "wrong" answers reveal about how students construct?

Student mistakes often reflect coherent but incomplete conceptions rather than random slips (Smith, diSessa, & Roschelle, 1993). A student who sees the number 14 and says it is "one-four" is seeing digits as separate labels, revealing a gap in the concept of place value. Similarly, a student who insists that $\frac{1}{8}$ is larger than $\frac{1}{4}$ because "8 is larger than 4" is logically overgeneralizing their whole-number knowledge. These errors are windows into how students construct mathematical meaning and are ripe for productive cognitive conflict.

- **What to Do Next:** Do NOT just say "no." Instead, use manipulatives like fraction strips and number lines to compare. Have students physically place $\frac{1}{4}$ on top of $\frac{1}{8}$ to see which is larger. Ask, "Would you rather have one piece of a sandwich cut into 4 pieces or one piece of a sandwich cut into 8 pieces?" This builds the understanding that the denominator represents the number of partitions, so a greater number of partitions yields a smaller unit (Boaler, 2016).

II. Building Conceptual Understanding

How does misunderstanding a core operation reveal deeper obstacles to mathematical reasoning?

A child's misunderstanding of an operation can be a major barrier to future learning. A student who sees $8 = 3 + \underline{\quad}$ and says, "This is backwards!" interprets the equals sign as an operator meaning "the answer is," not as a symbol of equivalence—a foundational barrier to algebraic thinking. Likewise, the belief that "division makes things smaller" leads a student to claim that $12 \div \frac{1}{2}$ must be 6, showing a lack of conceptual understanding of the operation.

- **What to Do Next:** Present equations in multiple formats: $8 = 5 + 3$, $8 = 8$, $2 + 6 = 5 + 3$. Use a bar model to show relational thinking and that both sides of the equal sign must represent the same quantity. For division by a fraction, use a real-world context: "How many half-cups of flour can you get from 12 full cups?" Use a bar model with $\frac{1}{2}$ units to show that division by a unit fraction is equivalent to multiplication by its reciprocal (National Research Council, 2001).

III. Developing Procedural Fluency Through Error Analysis

Are students truly understanding procedures, or are they merely memorizing steps without grasping their meaning?

Procedural errors are often systematic, revealing a flawed understanding of the underlying principles rather than just careless mistakes (Booth et al., 2013). When a student solves $52 - 16$ by taking the smaller digit from the larger in each column (writing 44), it reveals a rigid, non-regrouping understanding of "take away." Similarly, the "just add a zero" rule for multiplying by 10 is a fragile shortcut that fails with decimals, leading a student to solve 4.5×10 as 4.50, masking the conceptual understanding of increasing by a place value.

- **What to Do Next:** Use base-ten blocks to model regrouping. Show 52 as 5 tens and 2 ones. "We don't have enough ones to take away 6, so we must partition a unit of ten into 10 units of one. Now we have 4 tens and 12 ones. Now we can subtract." For decimals, use a place value chart to show that multiplying by 10 increases a digit by one place value. Show 4.5 (4 ones, 5 tenths) becoming 45 (4 tens, 5 ones).

IV. Fostering Mathematical Practices and a Collaborative Culture

How can analyzing mistakes foster deeper reasoning and classroom collaboration?

When the primary goal is the correct answer, mathematics can feel like a solitary, high-stakes pursuit. However, when the focus shifts to reasoning and sense-making, the classroom undergoes a transformation. Analyzing errors is a natural habitat for the Standards for Mathematical Practice, such as "constructing viable arguments and critiquing the reasoning of others" (National Research Council, 2001). A student who claims that "the order does not matter in division" provides a perfect opportunity for this kind of discourse.

- **What to Do Next:** Use a real-world context to make the meaning concrete. "If you have 12 cups of juice and 4 friends, how many cups does each friend get?" ($12 \div 4 = 3$). Then ask, "If you have 4 cups of juice and 12

friends, how much juice does each friend get?" ($4 \div 12 = \frac{1}{3}$). The context makes the non-commutative nature of the operation more apparent and invites students to critique the initial claim collaboratively.

V. Cultivating a Growth Mindset and Resilience

In what ways do classroom attitudes about mistakes and misconceptions shape mathematical confidence and perseverance?

The impact of an error-positive classroom extends far beyond cognitive gains; it shapes students' self-perceptions and learning attitudes. Research on mindset demonstrates that when students view intelligence as malleable, mistakes become opportunities for growth, rather than evidence of deficiency (Dweck, 2006). When learners recognize and reflect on errors, it primes the brain for deeper insight, a concept supported by theories of “productive failure” (Kapur, 2014). Working through confusion builds resilience and strengthens students' belief in their capacity to succeed, also known as self-efficacy (Bandura, 1997).

- **What to Do Next:** Model a positive, curious stance. Use language like, “That’s an interesting mistake—it shows you’re thinking! Let’s see where that idea leads.” Regularly incorporate routines where you celebrate an especially instructive error for the whole class to analyze and discuss. This shifts the classroom norm from “who is right?” to “what can we learn from this?”

Conclusion: Toward Confident, Flexible Mathematical Thinkers

Mistakes and misconceptions are not detours; they are the main road of mathematical learning. By examining errors as essential insights into student reasoning, educators foster conceptual depth, procedural flexibility, and resilience. The intentional practices of analyzing errors, using visual models, and grounding ideas in real-world contexts transform classrooms from places of correction to communities of reasoning and inquiry.

Ignoring errors is a missed opportunity; embracing them prepares students not just for tests, but for lifelong mathematical thinking. Shifting from rote tricks to rich, reflective practices nurtures the intellectual courage essential for success in and beyond the mathematics classroom (Boaler, 2016; Borasi, 1996; National Research Council, 2001).

References

- Bandura, A. (1997). *Self-efficacy: The exercise of control*. W.H. Freeman.
- Blackwell, L., Trzesniewski, K., & Dweck, C. S. (2007). Implicit theories of intelligence predict achievement across an adolescent transition: A longitudinal study and an intervention. *Child Development*, 78(1), 246–263.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. Jossey-Bass.
- Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*, 25, 24–34.
- Borasi, R. (1996). *Reconceiving mathematics instruction: A focus on errors*. Ablex Publishing.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

Kapur, M. (2014). Productive failure in learning math. *Cognitive Science*, 38(5), 1008–1022.

Kobiela, M., & Lehrer, R. (2019). Supporting students' progressive development of meaning of the minus sign. *ZDM*, 51(1), 139–152.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. J. Kilpatrick, J. Swafford, & B. Findell (Eds.). National Academies Press.

Smith, J. P., diSessa, A. A., & Roschelle, J. (1993). Misconceptions reconceived: A constructivist analysis of knowledge in transition. *The Journal of the Learning Sciences*, 3(2), 115–163.

Social Media

Are We Teaching Math for Right Answers—or for Resilient Thinkers?

It's efficient to correct student errors quickly and move on. But what if, in our rush to fix mistakes, we're missing the most powerful learning opportunities? The latest research in cognitive science and math education shows that mistakes are not setbacks—they are essential data points on the path to deep understanding.

If we want to cultivate students who are flexible, confident problem-solvers—not just answer-getters—we must reshape how we view and use errors in the classroom.

Our latest DMTI Insights, "**Embracing Mistakes and Misconceptions in Mathematics Education**," synthesizes the evidence and offers actionable strategies to make this shift:

- **Why conceptual understanding** is unlocked by analyzing *why* a student thought $\frac{1}{8}$ was larger than $\frac{1}{4}$, rather than just marking it wrong.
- **Powerful ways to use student errors** as springboards for cognitive conflict, building a growth mindset and procedural fluency simultaneously.
- **How visual models and real-world contexts**—like using bar models to understand division by fractions—help students rebuild flawed mental models into robust understanding.
- **Practical routines for the classroom** that transform the culture from "fear of being wrong" to "curiosity about thinking."

It's time to move beyond simply correcting mistakes—and toward building the intellectual courage that lasts a lifetime.

What's your favorite strategy for turning a student's "mistake" into a teachable moment for the whole class? Share your experiences in the comments below!



For more information, contact@dmtinstitute.com or follow us on social media:



facebook.com/dmtinstitute



instagram.com/dmtinstitute/



@DMTInstitute.bsky.social



mathsuccess.dmtinstitute.com



x.com/dmtinstitute



linkedin.com/company/dmt-institute



Tiktok.com/@math.success



youtube.com/@dmtinstitute

