

NEWSLETTER

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THE UNDERSTANDINGS THAT BUILD MULTIPLICATIVE THINKING

In today's edition, we explore:

- **The Classroom Reality:** Why a student who knows 6×8 can still be counting by ones — and why a correct product is not the same as reasoning about the unit
- **The Structure:** How students move from reasoning with a unit, to coordinating quantities, to recognizing multiplicative structure, and to comparing and scaling. This shift, not fact fluency, is what predicts later success
- **The Instructional Shift:** How to assess and teach for multiplicative thinking, and the habits that quietly keep it additive

Educators

Mrs. Alvarez asked her class what six times eight is. Nearly every hand went up with forty-eight. Then she asked them to model it, connect it to forty-eight divided by eight, and say what happens if both factors double—and the confident answers came apart.

One student reached forty-eight by adding eight, eight, eight, and on down the row. The product was right; the reasoning was still counting ones. Another read six times four as sixty-four, joining the two numbers into one. A third treated forty-eight divided by eight as a brand-new problem, unconnected to the six times eight she had just solved.

These students share something important: each produces a reasonable-looking move — a memorized fact, a running sum, an answer, but none treats a collection as a unit or holds two quantities together. A correct product is not the same as multiplicative thinking.

Our latest DMT Insight — *Multiplication and Division* — explains why the move from counting to multiplying is one of the strongest signs of where a grade 3–6 student stands, which understandings matter most, and how to assess them before the gap widens.

The Classroom Reality: When Multiplication Stays Additive

In many classrooms, multiplication and division arrive as procedures and facts rather than as ways of reasoning about units:

- Multiplication is introduced as repeated addition and left there, so the composite unit, the collection treated as one item, never forms
- The facts are memorized as isolated pairs, so 6×8 and $48 \div 8$ stay unrelated rather than one relationship

- The multi-digit algorithm is taught as steps before the area model gives it meaning, so “carry the one” has no structure behind it
- Division is taught months after multiplication, so sharing and measuring feel like new procedures rather than the other direction of the same reasoning

So students develop the logic their instruction has taught them:

- “To multiply, add the same number over and over.”
- “The facts are just things you memorize.”
- “Division is a separate thing you learn later.”

The answers look right—even when the reasoning behind them is missing.

The Structure: Why Multiplicative Thinking Predicts

Research from cognitive psychology and mathematics education points to a consistent finding: what carries students forward is not faster recall, but the ability to treat a collection as a unit and coordinate two quantities at once.

The critical shift happens when students stop counting ones and start reasoning with a composite unit. Seeing six as the unit and iterating that unit four times, rather than seeing twenty-four separate objects, is what separates students who are ready for fractions, ratios, and algebra from those who meet the same wall again and again (Steffe, 1992; Simon & Blume, 1994).

Multiplication and division are one relationship. Sharing twenty-four into six equal units and measuring how many sixes fit in twenty-four are the two types of division, and both are the inverse of multiplication. Taught together, division ceases to be a separate procedure and becomes the opposite reasoning (Vergnaud, 1988).

The distributive property connects known facts to the algorithm. An area model for seven times sixteen decomposes the sixteen into ten and six, and the partial products are what the standard algorithm records in shorthand. A student who builds this understanding does not memorize the rule; they understand the algorithm's structure (Hiebert & Carpenter, 1992).

Magnitude keeps a product honest. Knowing that six times forty is two hundred forty, not twenty-four, and being able to place it, keeps the factors tied to the result. The number line, used as a tool for reasoning rather than a counting device, makes that visible (Bruner, 1966).

The pattern is consistent: when multiplication is built on composite units and coordinated quantities rather than on counting and memorized steps, students reason about how quantities relate, and that reasoning holds, from whole numbers to fractions, proportion, and early algebra.

The Instructional Shift: Teach and Assess for Multiplicative Thinking

The DMT Insight proposes a simple shift: ask students to reason about the unit and the relationship before they compute. A few deliberate moves:

- **Build the composite unit; do not stop at repeated addition.** Put six counters in a bag and reason about four bags as four sixes, then move to bar models so the unit stays visible while the ones show inside it.

- **Use the area model for the algorithm.** Decompose or partition a factor, multiply the quantities, and recombine them, so the partial products hold the meaning that the standard algorithm steps compress.
- **Estimate before computing.** “About how large must this be, and where does it land on the number line?” keeps the factors tied to the result and catches incorrect answers.

When teachers ask, at every grade level, “Is this student treating a collection as a unit and coordinating two quantities, or counting and repeating addition?”, the difference between an additive habit and multiplicative reasoning comes into focus.

Small questions create large shifts:

- “What is the unit here, and how many iterations? Show me another way.”
- “How is 6×8 like $48 \div 8$? How could you use 6×5 to find 6×8 ?”
- “Now share the total. How large is each unit?”
- “Is one quantity more than the other, or how many times as large?”

These questions move students from counting objects toward reasoning with the unit.

Your Turn to Share

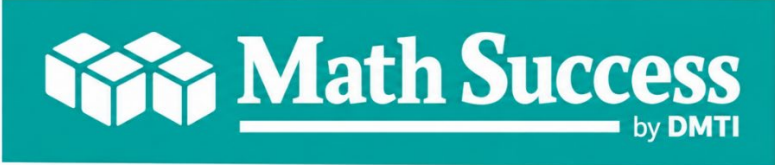
We would love to hear from you. What is one moment when a student’s reasoning about multiplication or division, or a slip back into counting, revealed an understanding or a gap that a correct answer alone would have hidden?

Share your thoughts at contact@dmtinstitute.com. We will compile and elevate your insights to help advocate for stronger teacher support across districts.

Keep inspiring mathematical minds!

More Information

This newsletter is just a preview — there is much more to explore. For the complete research overview, visit the DMT Insights page at dmtinstitute.com/research. You can also try a Math Learning Profile (MLP) diagnostic for grades 3-6 at mathsuccess.io.




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