

NEWSLETTER

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PROGRESSIVE FORMALIZATION: FROM MATHEMATICAL RELATIONSHIPS TO FORMAL PROCEDURES

How students connect quantities, representations, and procedures.

In today's edition, we explore:

- **Progressive formalization:** how mathematical ideas become more precise
- **Classroom tips:** connecting relationships and procedures
- **A worked example:** from partial products to the standard algorithm

Educators!

Progressive formalization describes how students develop mathematical relationships and connect them to formal procedures. Students identify units and quantities, represent relationships, and use mathematical properties to explain their calculations.

Consider 14×6 . Decomposing 14 into 10 and 4 gives two partial products, 60 and 24. An area model, ratio table, and standard multiplication algorithm can each represent this same relationship.

Classroom Tips

1. **Identify units and quantities.** Ask, "What does each number represent? What unit is being iterated?"
2. **Connect representations.** Ask students how the area model, ratio table, and algorithm represent the distributive property.
3. **Develop procedural fluency.** Teach procedures explicitly and provide feedback and practice that develop accuracy, efficiency, and flexibility.

From Partial Products to a Formal Algorithm

When students calculate 14×6 , they can decompose 14 into 10 and 4. This produces two partial products, 60 and 24, which compose the total of 84.

Partial products

$$\begin{array}{r} 14 \\ \times 6 \\ \hline 24 \\ + 60 \\ \hline 84 \end{array}$$

4 ones $\times 6 = 24$ ones
1 ten $\times 6 = 6$ tens, or 60

Standard algorithm

$$\begin{array}{r} 2 \\ 14 \\ \times 6 \\ \hline 84 \end{array}$$

24 ones = 2 tens and 4 ones:
write 4, regroup the 2 tens
6 tens + 2 tens = 8 tens

The 24 ones are recorded as 4 ones and 2 regrouped tens.
The 6 tens from 6×10 (the 60) and the 2 regrouped tens make 8 tens.

The standard multiplication algorithm organizes the same calculation using place value. Teachers can show how the 24 ones are regrouped as 2 tens and 4 ones, and how the 6 tens from 6×10 combine with the 2 regrouped tens to make 8 tens. With the 4 ones, the product is 84.

Students should understand the mathematical relationship and develop an efficient method for calculating the product.

What Does the Research Say?

Research on conceptual and procedural knowledge suggests that the two develop together, each supporting the other (Rittle-Johnson et al., 2015). Representations help when they are selected with attention to students' prior knowledge: a representation that helps one student may be unnecessary for another (Kalyuga et al., 2003).

Keep developing mathematical thinkers.

More Information

Read DMT Insight 30 and download the Companion at mathsuccess.io/dmtinsights.

To share a classroom example or a question, contact us at contact@dmtinstitute.com.

For more information on professional development and curricular resources, visit mathsuccess.io.



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