



Developing Mathematical Thinking:
Why Teacher Knowledge is the Key

DMTI INSIGHTS

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WHY TEACHER KNOWLEDGE IS THE KEY TO FIXING ELEMENTARY MATH

(DMT INSIGHTS)

Introduction

Why do we continue to see low math performance despite better standards, assessments, and curricula?

Across the nation, elementary math performance has remained stagnant for decades. Although schools have adopted clearer standards and higher-quality materials, a central issue remains unresolved: **teachers are not being adequately prepared or supported to teach mathematics for deep understanding**. Research consistently shows that the single most powerful school-based factor in student learning is the quality of instruction (Hattie, 2009). However, most teacher preparation programs and workshops fail to build teachers' mathematical knowledge, understanding of how children learn, or skill with high-leverage pedagogical practices (Ball et al., 2008; AMTE, 2017). As a result, teachers enter classrooms underprepared, and schools depend on professional development to rebuild the foundation that preparation programs should have provided.

Cognitive Foundations: Why Strong Teacher Knowledge Changes Student Thinking

What becomes possible in classrooms when teachers possess deep mathematical and pedagogical knowledge?

Effective math teaching requires teachers to understand **concepts, structures, and representations—not just procedures**. Research on Mathematical Knowledge for Teaching (MKT) demonstrates that teachers with strong content and pedagogical content knowledge significantly improve student outcomes (Hill et al., 2005). When teachers understand the “why” behind mathematical ideas, mathematics becomes coherent, and sensemaking becomes central. In contrast, teachers without deep preparation rely heavily on rules and demonstrations, which results in procedural knowledge that does not transfer. In this way, teacher preparation directly shapes the mathematical experiences available to students.

Structural Knowledge: The Mathematical Actions Teachers Must Understand

What must teachers understand about mathematical structure before they can help students learn it?

Students develop understanding through actions such as composing, decomposing, unitizing, iterating, and partitioning. Teachers must understand these actions deeply because **students cannot learn conceptual**

mathematics through procedures alone. However, many preparation programs treat these foundational ideas superficially (CBMS, 2012). Teachers often graduate without experience using number lines, area models, or manipulatives to build reasoning, and without tools for analyzing student thinking. Without a firm grasp of structure, teachers cannot diagnose misconceptions or connect big mathematical ideas across grades. This leaves students with a fragmented and unstable understanding.

How Students Learn Math: The Missing Psychology in Teacher Preparation

Why must teacher preparation include a deep understanding of the psychology of how children learn mathematics?

Most teacher preparation programs and professional learning workshops offer little to no experience in **how children actually learn math and develop mathematical understanding.** Learning-sciences research—on developmental trajectories, spatial reasoning, dual coding, working memory, and conceptual–procedural relationships—is rarely taught to future teachers (Clements & Sarama, 2014; Mix & Cheng, 2012; Paivio, 1986; Rittle-Johnson & Alibali, 1999). Teachers who understand how children learn can better interpret errors, anticipate misconceptions, and design tasks that build deep reasoning. However, because most programs neglect learning psychology, teachers enter classrooms without a clear sense of how children move from informal ideas to formal concepts. This gap limits their ability to support meaningful sensemaking.

University Preparation: Misalignment Between What Teachers Need and What Programs Provide

Why do universities consistently fall short in preparing teachers for real math instruction?

Most teacher preparation programs underemphasize mathematics and provide little professional development in the psychology of learning or practice-based pedagogy (NRC, 2001; AMTE, 2017). Elementary majors often take only one or two courses that resemble liberal arts math rather than mathematics for teaching. This leaves future teachers without **the content and pedagogical foundation required for effective instruction.** Compounding this, math and education faculty often work in isolation from each other, leading to programs in which content and methods are disconnected. Candidates rarely practice teaching routines, analyze misconceptions, or build representational fluency. As a result, teachers graduate without the knowledge needed to support their students' conceptual understanding.

Instructional Consequences: How Preparation Gaps Affect Classrooms

What does instruction look like when teachers lack deep preparation in math, psychology, and pedagogy?

When teachers lack foundational preparation, instruction defaults to rules such as “cross-multiply,” “stack the numbers,” or “move the decimal.” Students learn isolated steps but do not build conceptual understanding. The result is **procedural fluency without meaning**, which collapses when students encounter more complex ideas such as fractions or algebra. Teachers who lack preparation may use manipulatives or models incorrectly, causing students to see them as add-ons rather than thinking tools. These gaps disproportionately harm students in communities served by uncertified or fast-track teachers (von Hippel et al., 2018). In this way, inadequate preparation becomes an equity issue.

Why Typical Workshops Fail

Why are traditional workshops ineffective at improving math instruction?

One to two-day workshops provide exposure, not transformation. Research shows that such professional development (PD) rarely improves practice because **teachers need sustained learning, not isolated strategies**, to change instruction (Garet et al., 2001). Workshops do not build conceptual content knowledge, misunderstanding analysis, or pedagogical fluency. They often replicate the weaknesses of teacher preparation—short, disconnected, and removed from classroom practice. As a result, schools spend valuable time and money on PD that produces little change in student learning.

High-Quality Professional Development: Rebuilding What Teacher Preparation Did Not Provide

How does sustained, content-rich PD transform teacher practice and student learning?

High-quality professional development builds teacher expertise across mathematics, learning psychology, and pedagogy. Effective PD is long-term, content-focused, and grounded in models, representations, and student thinking (Desimone, 2009). It provides teachers with opportunities to rehearse instructional routines, analyze errors, and apply new practices in their classrooms. The most effective programs also promote a consistent **structural language**—such as unit, partition, iterate, decompose, compose, and equal—which anchors reasoning and creates coherence across lessons. When teachers build deep knowledge and pedagogical skill, classrooms shift from memorizing steps to mathematical sensemaking.

Conclusion

Elementary mathematics success depends on teachers' knowledge, not just on curriculum materials. Strong preparation programs and sustained PD must build teachers' **conceptual understanding, learning-science knowledge, and pedagogical skills**. When teachers have strong foundations, classrooms become places where students reason, model, explain, and connect ideas. Improving teacher knowledge is not just best practice—it is an equity imperative. The path forward is clear: meaningful improvement in math requires meaningful investment in the adults who teach mathematics.

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Social Media

So, why aren't expensive curriculum adoptions moving the needle in elementary math achievement?

Our newest DMT Insight breaks down the real story behind stalled results: **it's not the materials—it's the missing professional learning.**

- **The “Why”:** Learn why curriculum alone produces only modest gains, while teachers with strong mathematical understanding and knowledge of how students learn create the *true* impact on student achievement.
- **The “How”:** See how sustained, math-specific PD helps teachers use models, representations, and discourse routines the way curricula *intend*—transforming materials from scripts into tools for reasoning.
- **The “What Now”:** Explore a practical model for balancing investments: curriculum as the roadmap, and professional development as the engine that brings it to life.

It's time to stop expecting materials to solve a human-capital problem. Invest in teachers!

What is the biggest challenge your school or district faces in helping teachers learn math for teaching? Share your insights below!

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