



Developing Mathematical Thinking: Drills vs. Strategies

DMTI INSIGHTS

BY

Jonathan Brendefur, PhD
and Associates

DRILLS VS. STRATEGIES: BUILDING FLEXIBLE MATHEMATICAL THINKERS

(DMT INSIGHTS)

Introduction

In K-8 mathematics, fluency with basic addition and multiplication facts is a non-negotiable foundation. However, a significant gap exists between mere memorization and true proficiency. We see this starkly in the data: while many 3rd graders can recall their math facts, **by 8th grade, only about 17% of students have maintained this fluency**. This dramatic drop-off reveals that short-term recall is not the same as long-term mastery. The ultimate goal is to create flexible thinkers and problem solvers who can compose, decompose, and reason with numbers, not just recall them quickly (Baroody, 2006; Brendefur & Strother, 2015).

Despite this, many instructional programs default to drill-heavy approaches—such as timed tests, repetitive flashcards, and massed practice. While these can improve short-term speed, they consistently fail to develop the conceptual understanding and strategic competence necessary for long-term success and application (Boaler, 2014).

This DMT Insight synthesizes research from cognitive psychology and mathematics education to argue that an integrated approach—combining explicit strategy instruction rooted in **structural language** (*unit, compose, decompose, iterate, partition, equal*) with purposeful retrieval practice—is the most effective path to building durable, flexible, and transferable fact fluency.

Redefining Fluency: Automaticity Rooted in Understanding

What if being 'fast' at math is actually slowing your students down?

True fluency is more than speed and accuracy; it is the efficient, flexible, and appropriate application of facts in problem-solving contexts (NRC, 2001). This requires a dual perspective:

From Cognitive Psychology: Automatic fact retrieval frees up limited working memory for higher-order tasks (Sweller et al., 2011). However, this automaticity is fragile if it is built solely on rote memorization. Durable recall depends on facts being embedded in rich, interconnected networks of meaning (Bransford et al., 2000).

Mathematics Education: Fluency involves the ability to deconstruct and reconstruct numbers using strategies like making ten, doubling, iterating, and partitioning (e.g., seeing 6×7 as $(5 \times 7) + (1 \times 7)$). Brendefur & Strother (2015)

crucially distinguish between *fluency* (fast, accurate recall) and *flexibility* (the ability to derive facts using reasoning), noting that the latter is a prerequisite for robust, long-term mastery of the former.

Thus, we must redefine fluency as **automatic retrieval, strategic flexibility, and conceptual understanding**.

The Shortfalls of Drill-Only Approaches

Why do students who ace their timed tests in 3rd grade often fail word problems and forget them over the next few years?

Programs relying solely on massed drills and timed tests exhibit several documented shortcomings:

Limited Transfer and Brittle Knowledge: Students may pass a fact test but be unable to apply those facts in novel problems. Strategy instruction, not drill, leads to improved performance on transfer tasks (Baroody, 2006).

Inhibition of Strategic Flexibility: Drill-centric practice teaches that there is one "right" way to get an answer—quickly. It does not foster the adaptive, multi-strategy reasoning required for complex problem-solving.

Vulnerability to Interference: Similar facts (e.g., 6×7 , 7×6 , 6×8) compete and cause confusion. Drills do not help students build cognitive networks to suppress this interference, leading to shallow encoding and forgetting.

Induction of Math Anxiety: The focus on speed creates anxiety, which consumes the very working memory capacity that automaticity is meant to free up (Ramirez et al., 2018).

Equity Concerns: Rigid, speed-based approaches disproportionately harm students with learning differences, executive functioning challenges, or math anxiety, widening achievement and confidence gaps (Boaler, 2016).

Evidence in Action: A study by Brendefur et al. (2015) found that after a 5-week intervention, students in a strategy-based program gained an average of 6.08 correct facts per minute, compared to only 0.79 facts for students in a drill-only program—a nearly 8-fold difference in efficacy.

An Evidence-Based Model: Strategy First, Then Retrieval

What's the secret to making math facts 'stick' for the long term?

The most effective model is a phased, integrated approach that builds conceptual understanding before demanding automaticity.

Phase 1: Build Strategic Understanding. Explicitly teach strategies using structural language (unit, compose, decompose, partition, iterate, equal) and progress through representations (visual → abstract) to build deep conceptual schemas. Teachers should model this language aloud (“I decomposed 14 into 10 and 4,” or “I partitioned 12 into three equal groups”) to strengthen students’ structural awareness.

Phase 2: Implement Purposeful Retrieval Practice. Once strategies are understood, use spaced and varied retrieval practice to strengthen recall pathways. This practice is meaningful because it reinforces connected knowledge.

Interleaving facts of different operations (addition, subtraction, multiplication, and division) requires students to choose appropriate strategies and enhances long-term retention (Rohrer & Taylor, 2007).

Phase 3: Foster Flexible Transfer. Embed fact use in complex problems and encourage metacognition (“How did you solve it? Solve it using a different strategy.”) to promote adaptive reasoning. For example, if a student knows $6 \times 7 = 42$, teachers might ask, “How could you use that to find 6×8 ?” This type of question builds relational connections among facts.

This model ensures that speed is built upon a foundation of sense-making, resulting in fluency that is both durable and flexible.

Conclusion

The evidence is clear: drill-only programs are an inefficient and often counterproductive method for building the mathematical thinkers our students need to become. To cultivate true mathematical proficiency, we must intentionally redesign fluency instruction to focus on structure and strategy.

For educators, coaches, and parents, this means:

- Prioritizing strategy instruction and structural language (unit, decompose, compose, iterate, partition, and equal)
- Replacing high-stakes timed tests with low-stakes retrieval games and rich discussions like Number Talks.
- Celebrating strategic thinking and reasoning as much as, if not more than, speed.

By integrating strategic reasoning with thoughtful practice, we move beyond creating "drill masters" and instead empower all students as confident, capable, and flexible mathematicians.

References

- Baroody, A. J. (2006). Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, 13(1), 22–31.
- Bjork, R. A. (1994). Memory and metamemory considerations in the training of human beings. In J. Metcalfe & A. Shimamura (Eds.), *Metacognition: Knowing about knowing* (pp. 185–205). MIT Press.
- Boaler, J. (2016). *Mathematical mindsets: Unleashing students’ potential through creative math, inspiring messages, and innovative teaching*. Jossey-Bass.
- Bransford, J. D., Brown, A. L., & Cocking, R. R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school*. National Academy Press.
- Brendefur, J., & Strother, S. (2015). Developing multiplication fact fluency. *Advances in Social Sciences Research Journal*, 2(10), 166–176. <https://doi.org/10.14738/assrj.210.166>
- Brendefur, J., & Strother, S. (2021). *Math facts: Kids need them. Here’s how to teach them*. Developing Mathematical

Thinking Institute.

Dweck, C. S. (2006). *Mindset: The new psychology of success*. Random House.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Ramirez, G., Chang, H., Maloney, E. A., Levine, S. C., & Beilock, S. L. (2018). On the relationship between math anxiety and math achievement in early elementary school: The role of problem-solving strategies. *Journal of Experimental Child Psychology*, 167, 404–414.

Rohrer, D., & Taylor, K. (2007). The shuffling of mathematics problems improves learning. *Instructional Science*, 35(6), 481–498.

Sweller, J., Ayres, P., & Kalyuga, S. (2011). *Cognitive load theory*. Springer.

Social Media

So, what is the secret to building fluency that lasts a lifetime, not just until the next test?

Our latest DMT Insight reveals the decisive shift from **Drills to Strategies**:

- **The "Why"**: Discover why strategy-based instruction leads to **8x greater gains** in fact fluency compared to drill-only practice, creating durable, flexible knowledge.
- **The "How"**: Learn how using **structural language** (like *decompose and compose*) builds the neural networks that prevent forgetting and enable transfer.
- **The "What Now"**: Get a clear, 3-phase model (Strategy First, Then Purposeful Retrieval) to ensure speed is built on a foundation of sense-making.

Stop the 3rd-to-8th-grade slide. It is time to replace short-term drills with long-term thinkers.

What is the biggest challenge you face in helping students *truly* retain a solid understanding of math? Share your experience below!



For more information, contact@dmtinstitute.com or follow us on social media:



facebook.com/dmtinstitute



instagram.com/dmtinstitute/



@DMTInstitute.bsky.social



mathsuccess.dmtinstitute.com



x.com/dmtinstitute



linkedin.com/company/dmt-institute



Tiktok.com/@math.success



youtube.com/@dmtinstitute

