

DMT INSIGHTS

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LINEAR MODELS VS. 100S CHARTS: WHAT DO STUDENTS SEE?

Bar models, number lines, and the role of the hundreds chart

Why Linear Representations Matter

Bar models, number lines, and hundreds charts all organize numbers, but each represents a different aspect of mathematics. A bar model represents quantities and relationships through lengths, partitions, and comparisons. A number line represents numerical position, magnitude, distance, and equal intervals. A hundreds chart organizes the counting sequence into a two-dimensional grid, making counting and base-ten patterns visible.

DMTI recommends introducing linear models early, using bar models as a primary representation for number relationships and operations, and continuing to use them through elementary mathematics and into algebra. Hundreds charts have a narrower role: counting sequences and number patterns.

Bar models and number lines organize quantities along one dimension. A bar model uses length to represent quantity, while a number line uses equal intervals to represent numerical distance. A hundreds chart arranges numbers in rows and columns. This arrangement helps with counting and recognizing patterns, but the physical distance between two numbers on the chart does not consistently represent their numerical difference. For addition and subtraction, DMTI recommends linear models because they make the quantities and their relationships more explicit. The cognitive demands of a representation also depend on the task, its design, and students' familiarity with it.

The hundreds chart is a two-dimensional layout, but it does not represent two independent numerical dimensions like a coordinate plane. A bar model is drawn on a page but uses length as its primary quantitative dimension.

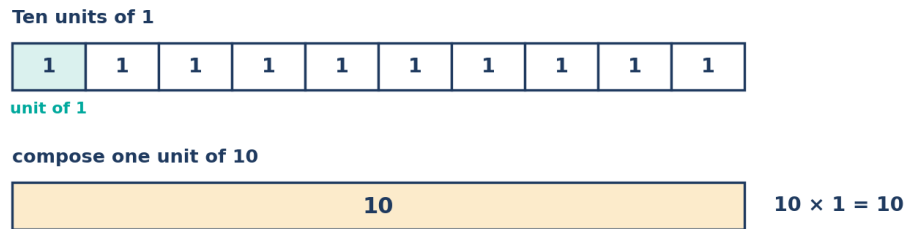
Key Idea • A bar model represents quantities and relationships. A number line represents position and distance. A hundreds chart organizes the counting sequence.

The Bar Model: Units, Magnitude, and Relationships

A bar model uses length as its primary quantitative dimension. Each part's length represents a quantity, and the parts compose the total. Because the parts and the total appear together, students can compare magnitudes and examine how the quantities are related. Research on the model method used in Singapore describes how bar models represent part-whole and comparison relationships, support arithmetic word-problem solving, and connect to algebra (Ng & Lee, 2009).

Composing and Decomposing Ten

Ten equal units of 1 can be composed into one unit of 10 (Figure 1). The segmented bar shows both at once: ten units of 1 and one unit of 10 have the same length. Students can also decompose a ten into ones when a calculation requires it.

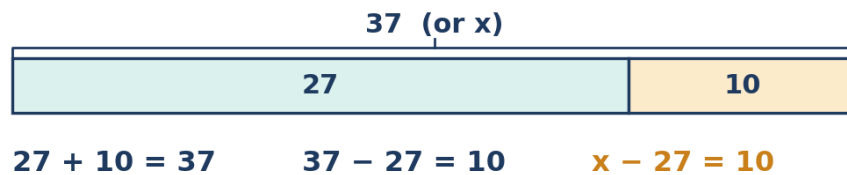


The same length represents ten units of 1 and one unit of 10.

Figure 1. Ten units of 1 compose one unit of 10.

Addition and Subtraction Through Bar Models

In Figure 2, a bar of 37 is partitioned into parts of 27 and 10, and the bracket shows the total. The same model represents $27 + 10 = 37$ and $37 - 27 = 10$: the parts compose the total, and subtracting one part from the total gives the other part. If the total is unknown, the same relationship can be represented by $x - 27 = 10$.



Lengths are proportional: the part of 10 is $10/37$ of the total length.

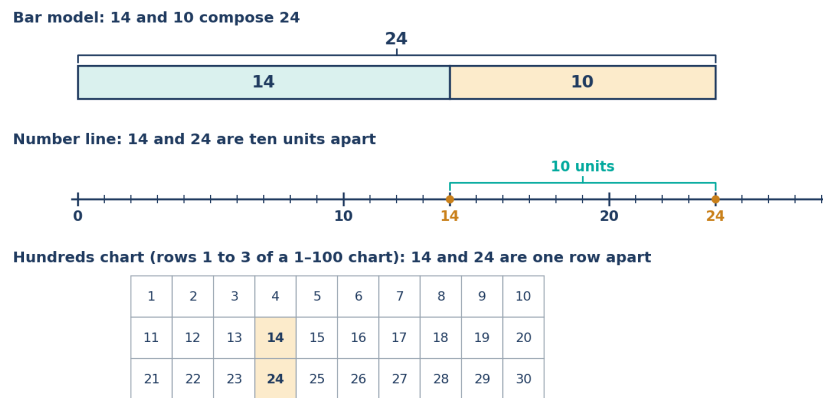
Figure 2. One bar model for $27 + 10 = 37$, $37 - 27 = 10$, and $x - 27 = 10$.

DMTI recommends that students represent and reason about the quantities rather than move across a grid to obtain answers.

Key Idea • Teach students to represent and reason about quantities, not to navigate a grid to obtain answers.

One Sum, Three Representations: $14 + 10 = 24$

Figure 3 shows $14 + 10 = 24$ as a bar model, on a number line, and on a hundreds chart. On a hundreds chart, 14 and 24 are one row apart. Students may learn that moving down one row increases a number by ten. The chart makes that pattern visible, but it does not directly represent ten as a length or show how 14 and 10 compose 24.



The bar model and the number line use the same scale. The parts 14 and 10 are proportional to the total of 24, and the interval from 14 to 24 is ten equal units. On the chart, the cells show a base-ten pattern but not these lengths.

Figure 3. $14 + 10 = 24$ as a bar model, on a number line, and on a hundreds chart.

A bar model represents 14 and 10 as quantities whose lengths compose the total of 24. The relative lengths represent their magnitudes. A number line shows that ten equal units separate 14 and 24. Both linear representations make the quantitative relationship explicit. DMTI recommends using bar models and number lines for addition and subtraction, while reserving hundreds charts primarily for counting forward and backward and examining number patterns.

Number Lines: Position, Distance, and Fractional Units

A number line represents numerical position, magnitude, distance, and equal intervals. The number 37 is located 37 units from zero, 7 units past 30, and 3 units before 40 (Figure 4). Because a number line includes zero, distance from zero represents magnitude (Siegler & Braithwaite, 2017). Students' accuracy in placing numbers on a number line is associated with broader mathematical competence: a meta-analysis of 263 effect sizes from 10,576 participants found a mean correlation of $r = .443$ (Schneider et al., 2018). This evidence is correlational and does not establish a particular teaching sequence.

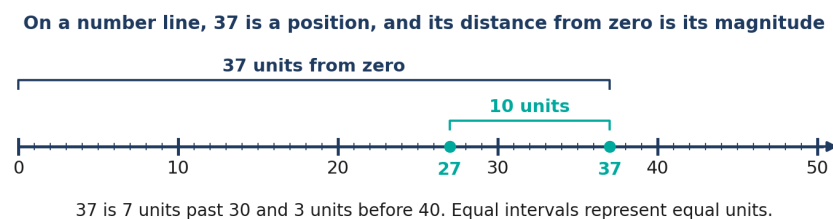


Figure 4. Thirty-seven is located by its distance from zero and from benchmarks.

Number lines also represent fractional units. Partitioning the unit of 1, the interval from 0 to 1, into four equal intervals and iterating one-fourth three times locates three-fourths. In a pilot study, a number-line intervention for struggling fifth graders was associated with gains in fraction understanding (Schumacher et al., 2018). Embodied number-line

training, in which children walked along a physical number line, has also improved numerical skills (Link et al., 2013). The empty number line, marked only where students need it, supports flexible addition and subtraction strategies (van den Heuvel-Panhuizen, 2008).

Hundreds Charts: Counting Patterns and Limited Instructional Use

A hundreds chart organizes the counting sequence in rows of ten. It is useful for counting forward and backward and for examining base-ten patterns, such as the ones digit staying the same down a column. In one study, the spatial layout of a hundreds chart changed how often kindergarten and first-grade children counted by tens, but had no measured effect on their place-value knowledge (Osana et al., 2023). DMTI recommends limiting the hundreds chart to counting sequences and patterns and not using chart movements as the primary method for addition and subtraction. This is an instructional recommendation, not a finding from a direct comparison of the two representations.

Figure 5 shows three counting patterns: counting forward from 14 to 15, 16, and 17; counting backward from 24 to 23, 22, and 21; and counting by tens, 14, 24, 34, 44. Students can investigate why the numbers in a column share the same ones digit: each row holds ten numbers, so each number is ten more than the number above it.



Rows 1 to 5 of a 1-100 chart. In the third chart, every number in the outlined column has 4 ones: each row holds ten numbers.

Figure 5. Counting forward, counting backward, and counting by tens on a hundreds chart.

The chart shows that 14 and 24 are one row apart. It does not directly show the relative lengths of 14 and 10 or how those quantities compose 24. A bar model makes that relationship visible, and a number line represents the ten-unit distance between the numbers (Figure 3).

Extending Bar Models from Elementary Mathematics to Algebra

Students can use the same bar-model structure to examine increasingly complex mathematical relationships.

Application	Example	Relationship
Early elementary	$7 + 3 = 10$	Compose parts into a total
Elementary	$37 - 27 = 10$	Compare quantities and determine the difference
Multiplication	$3 \times 4 = 12$	Three iterations of the unit of 4
Fractions	$3/4 = 1/4 + 1/4 + 1/4$	Partition the unit of 1 and iterate one-fourth
Algebra	$3x + 4 = 19$	Represent an unknown unit and solve for its value

In $3x + 4 = 19$, the bar shows three iterations of an unknown unit and a part of 4 composing 19. Removing the 4 leaves three iterations equal to 15, so each iteration of the unit is 5.

Research and Instructional Implications

Research supports several parts of this approach. Bar models represent part-whole and comparison relationships and connect arithmetic to algebra (Ng & Lee, 2009). Number-line estimation is associated with mathematical competence (Schneider et al., 2018). A representation's design influences how children use it (Osana et al., 2023). Research linking numerical cognition and mathematics education emphasizes magnitude and number relationships in early instruction (Hawes et al., 2021). DMTI recommends bar models over hundreds-chart movements for addition and subtraction, grounded in the mathematics these representations make visible.

Summary

DMTI recommends introducing bar models early and using them across grade levels to represent quantities, compose and decompose units, compare magnitudes, and examine mathematical relationships. Number lines extend this work through position, distance, partitioning, and iteration. Hundreds charts have a more limited role in counting and examining number patterns.

Representations support learning when instruction makes the relationships explicit. Explicit instruction, feedback, and practice help students develop conceptual understanding and procedural fluency together.

References

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Social Media

What does a bar model show that a 100s chart does not?

A bar model represents quantities and their relationships through length. A number line represents position and distance. A hundreds chart organizes the counting sequence. This DMT Insight examines:

- What each representation makes visible
- How $14 + 10 = 24$ looks as a bar model, on a number line, and on a hundreds chart
- How the same bar model extends to multiplication, fractions, and algebra
- What the research supports, and what is DMTI's instructional recommendation

Read the full Insight at mathsuccess.io/dmtinsights.

When your students add or subtract, are they reasoning about quantities or following a movement?

