DMT INSIGHTS

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Math Success by DMTI

MATH MODELS THAT MATTER

Introduction

A student can compute 503 − 297 correctly, line up the digits, regroup across the zero, and arrive at 206, yet still have little sense of the size of the answer. The procedure is correct, but the student may not understand the quantities or their relationship. This is one reason the choice of model matters.

Students do not learn mathematics only by hearing words or memorizing symbols. They develop mental representations of quantities, structures, and relationships that help them make sense of symbols. The model a student uses can strengthen that understanding, but only if it represents the mathematics we want students to notice and learn. Some models work well with small quantities and early ideas but quickly become useless as numbers and relationships become more complex. The important question is not whether a model works for one lesson, but which mathematical models are durable and will grow with students and the mathematics over time.

CLASSROOM SNAPSHOT

Teacher: How much is 503 minus 297?

Maya: 206. I lined up the digits and regrouped.

Teacher: About how large should the answer be?

Maya: I'm not sure. I'd have to do the steps again.

Diego: A little more than 200. 297 is close to 300, and 300 to 503 is about 203.

Teacher: How did you see that?

Diego: I thought about how far apart they are on the number line.

Maya's answer is correct. But Diego is reasoning about the magnitude of the numbers and the distance between them. The number line supports that reasoning.

Key Idea • A correct answer can hide a missing sense of quantity. The model a student reasons on decides whether number sense is built, and a weak model hides understanding.

Organizing a Quantity, or Preserving It: The Shift a Model Asks

Visual models are critical to understanding and do different kinds of mathematical work. A ten frame is limited to organizing small quantities around five and ten, and a number bond shows how quantities can be composed into a total. However, there are other models that are more powerful and represent additional relationships. A bar model uses length to represent relative quantity, and a number line represents magnitude, distance, order, and measurement through position, both of which build a deep understanding of mathematics and can be used at every level of mathematics.

An early model may help students organize a small quantity. A more durable model also supports reasoning about magnitude, distance, units, and how quantities change in relation to one another. Dual coding theory suggests that learning is strengthened when verbal and visual representations work together (Clark & Paivio, 1991). But the visual representation still needs to show the mathematical structure students are expected to understand.

> **Key Idea • Some models organize a quantity; others preserve its magnitude, distance, and relationship. The shift is in what the model asks the student to reason about.**

Iconic and Symbolic: What a Durable Model Must Hold

A durable model should represent several important mathematical ideas. It should show magnitude, so students can reason about how large a quantity is; distance, so they can compare how far apart quantities are; and the unit, so quantities can be composed through iteration and decomposed through partitioning. It should also support comparison and proportional reasoning. Models that represent these ideas can be used across a wider range of mathematics.

Models also differ in what they represent. Bruner (1966) distinguished iconic representations, which represent aspects of a quantity, from symbolic representations, which rely more heavily on convention. A ten frame, bar model, and number line provide visual information about quantity. A number bond is closer to a symbolic representation because the circles themselves do not show magnitude. A circle labeled 3 can be the same size as one labeled 30. Both models can show composition, but they do not provide students with the same quantitative information.

> **Key Idea • A durable model holds magnitude, distance, the unit, comparison, and proportion. An iconic model shows a quantity; a symbolic one only stands for it, and that difference sets their reach.**

Two Pathways: Why the Right Model Lightens the Load

From a cognitive psychology perspective, a model can reduce what students have to hold in working memory. Dual coding provides both visual-spatial and symbolic representations, so a position on a number line and its numeral can support the same idea (Clark & Paivio, 1991). When order, distance, and scale are visible in the model, students can devote more attention to the mathematical relationship they are trying to understand.

A consistent model can also help students develop a schema across related ideas. For example, aligned representations can help students see 3/4, 0.75, and 75% as different representations of the same magnitude rather than as three unrelated procedures. From a mathematics education perspective, this is important because students can continue to use the model to reason as the content becomes more complex. The model is useful when it makes the underlying mathematical structure visible. So, a ten-frame model is limited to numbers ten or less, and a number bond can help with composing and decomposing but also can lead students to misconceptions because the model does not help with magnitude or proportion. The number line and bar model are much more durable over time.

> **Key Idea • The right model gives a quantity two pathways and holds its structure on the page. Working memory is freed, and a stable schema can form.**

Low Ceiling and High Ceiling: Where Each Model Stops

Using these criteria, the four common models have different levels of reach. Ten frames and number bonds have a relatively low ceiling. Bar models and number lines have a higher ceiling because they continue to represent important relationships as the mathematics becomes more complex. This is a difference in purpose and reach, not simply a judgment that one model is good and another is bad.

Ten frames are useful for early number. They support subitizing, recognizing a small quantity without counting each item, and help students use five and ten as benchmarks. Clements (1999) describes the importance of seeing small quantities as organized groups rather than counting each item. The limitation is that a ten frame is fixed at ten and uses discrete cells. It can show that 8 is two less than 10, but it does not directly represent distance, scale, or proportional relationships such as the connection among 8/10, 0.8, and 80%.

Number bonds show the composition and decomposition of a total and can be useful for reasoning about a missing quantity. Their limitation is that the common circle-and-line format does not represent magnitude. A circle labeled 3 looks the same size as a circle labeled 30, so the quantitative information comes from the numerals rather than the model itself. Number bonds can organize part-total relationships, but they provide little information about distance, scale, or proportion. There is also very little research examining number bonds as a distinct instructional model.

Bar models represent quantity with length. This allows students to use the same model for total-and-quantity relationships, comparison, equal groups, and ratio. The structure of the situation can be represented before students choose an operation. Ng and Lee (2009) found that bar diagrams supported children in representing and solving algebraic word problems before they used formal symbolic algebra. This is one reason the model has greater reach: the same basic representation can support whole-number comparison in elementary school and more complex relationships in later grades.

Number lines have the strongest research base of the four. Siegler, Thompson, and Schneider (2011) place knowledge of numerical magnitude at the center of numerical development and describe the number line as a single structure that encodes whole numbers, fractions, decimals, and negative numbers as positions along a continuous magnitude. The National Mathematics Advisory Panel (2008) identified proficiency with fractions as foundational for algebra and highlighted the need to locate and compare fractions, decimals, and percents by magnitude. Resnick, Newcombe, and Goldwater (2023) found that reasoning about fraction and decimal magnitudes accounted for the relation between proportional reasoning and computation, and they read their results as supporting measurement models and spatial scaling. In the largest test of the idea to date, Nuraydin, Stricker, Ugen, Martin, and Schneider (2023) assessed nearly an entire national cohort of ninth-graders (N = 6,484) on a single number-line estimation task and found accuracy strongly associated with a standardized measure of mathematical achievement, with placement of fractions more closely related to achievement than placement of whole numbers.

> **Key Idea • Ten frames and number bonds organize early number but hold no magnitude. Bar models and number lines encode magnitude, units, comparison, and proportion, which is why they keep working.**

A Time-Limited Job: A Progression That Scales

A strong progression does not require abandoning ten frames and number bonds. It means using them for the ideas they represent well and then moving students toward models with greater mathematical reach. Ten frames are particularly useful for subitizing and benchmark reasoning around five and ten, while number bonds can support early composition and decomposition. As students begin reasoning more explicitly about magnitude, comparison, units, fractions, decimals, multiplication, division, and ratio, bar models and number lines should take a more central role. Double number lines, ratio bars, area models, and equations can then extend this reasoning into middle school and beyond.

There is also a mismatch between common classroom use and the research base. Number lines have a substantial research base, and bar models have research supporting their use for representing mathematical relationships. Ten frames and especially number bonds have a more limited dedicated research base. Lesh, Post, and Behr (1987) emphasize the importance of students moving among representations and translating between them. For that work to be productive, the representations need to make important mathematical relationships visible.

> **Key Idea • Give the early models a short, defined job, and give bar models and number lines the central, sustained role. The durable models are the least used and the best supported.**

Conclusion

The four models are not interchangeable. Ten frames and number bonds are useful for particular early ideas, but they do not directly represent magnitude, distance, scale, or proportional relationships. Bar models and number lines illustrate this structure more clearly. They can support reasoning about quantity, units, comparison, iteration, partitioning, and scaling across a broader range of mathematical concepts.

For teachers, coaches, and leaders, the implication is straightforward: choose a model based on the mathematical ideas it represents. Ask what students can see and reason about with the model, what the model does not show, and whether it will continue to be useful as the mathematics develops. From this perspective, number lines and bar models should have a central role from early number through fractions, proportional reasoning, and algebra.

> **Key Idea • Choose models for what they hold. The number line and the bar model preserve magnitude and relationships from early number to algebra, and that is the structure on which a coherent pathway is built.**

References

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Social Media

Not all math models deserve the same amount of instructional time.

Ten frames and number bonds can be useful tools in early number work. They can help young students see small quantities, compose and decompose numbers, and make basic part–whole relationships visible.

But they have limits.

As numbers become larger and students encounter subtraction, fractions, decimals, ratios, and algebraic relationships, those models often stop revealing the mathematics students need to see. A ten frame is constrained by its fixed structure. A number bond can show that a quantity is made of parts, but it does not show magnitude, distance, comparison, or the size of a relationship.

That is why the **number line** and the **bar model** should play the central, sustained role in mathematics instruction.

The number line makes number magnitude visible. It represents:

- The size and order of numbers
- Distance and difference
- Benchmark numbers and estimation
- Fractions and decimals as numbers with locations and size
- Integers and distance from zero
- Multiplicative and proportional relationships

The bar model makes relationships between quantities visible. It supports students in representing:

- Parts and wholes
- Comparisons between quantities
- Unknown quantities
- Multiplication and division situations
- Fractions, ratios, and percent
- Equations and early algebraic reasoning

These are not just elementary-school visuals. They are durable models that grow with students. They offer a coherent visual language for reasoning across grades rather than a collection of disconnected tools for individual lessons.

Our new **DMT Insight** explains why schools should be intentional about which models become central in the curriculum.

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[Classroom photograph]