DMT INSIGHTS

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Math Success by DMTI

GEOMETRIC MEASUREMENT: BEFORE THE FORMULAS

Introduction

A second grader reports a pencil as six inches long, a fourth grader gives the area of a rectangle as eight times five, and a sixth grader finds the volume of a box as four times three times two. All three answers can be correct, but the answers alone do not tell us how the students reasoned. Measurement is a count of equal-sized units that fill a quantity with no gaps or overlaps, and the number reported tells how many of those units the quantity contains. A fourth grader is shown a rectangle with the top row and left column of unit squares drawn and the rest of the region blank, and asked how many squares will cover it. She counts the squares she can see and writes 7. Asked to finish drawing them, she fills the region row by row and writes 12. Nothing about her arithmetic changed. What changed is that she built the array — and the array, not the count, is what the formula operates on. Measurement connects number and space. Before students can measure length, area, or volume, they must organize that quantity into equal units they can count. Battista and colleagues call this spatial structuring, and it develops gradually across grades 2–8 (Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998; Barrett, Clements, & Sarama, 2017).

> **Key Idea • A correct measurement can be produced without the understanding it appears to demonstrate. The number reports a count of units, and the units have to be there.**

The Geometric Measurement Math Learning Profile: Four Developmental Understandings

The Geometric Measurement Math Learning Profile (MLP) is organized around four developmental understandings: Unitizing, Measurement Magnitude, Spatial Structure, and Relational Thinking. Together they describe how students establish a unit, reason about the magnitude of the attribute being measured, organize units spatially, and coordinate related measures. Unitizing is establishing an equal-sized unit, holding its size constant, and iterating it to compose a quantity. Measurement Magnitude is understanding a measure as the amount of an attribute built from equal units, coordinating a starting point with accumulated intervals and reasoning from benchmarks. Spatial Structure is organizing a measurable region into equal units and the relationships among those units. Relational Thinking coordinates area and perimeter as distinct measures and reasons about what remains invariant and what changes. The sections that follow develop each of these in turn.

Placing the construct this way also sets its boundaries. Place Value develops the structure of units of ten. The magnitude of a fraction is developed in Fraction Concepts. Judgments about how two quantities scale together are developed in Scaling and Comparing. This unit focuses on measuring a spatial attribute: establishing the unit, accumulating it from an origin, organizing units within a figure, and coordinating two measures of the same figure.

> **Key Idea • Four developmental understandings organize geometric measurement: Unitizing, Measurement Magnitude, Spatial Structure, and Relational Thinking. Strategy Flexibility cuts across all four and is reported separately.**

What Measurement Asks of Students: Counting Equal Units That Fill a Quantity

To measure, students select a unit, iterate that unit across a quantity, and report how many units fit. This gives measurement a multiplicative structure from the beginning: a unit is repeated a certain number of times to compose the quantity. Research on measurement development identifies several understandings students must construct: the units must be equal in size, they must cover or fill the quantity with no gaps or overlaps, measurement begins from an origin, measures are additive and conserved when a figure is decomposed and rearranged, larger units produce smaller counts, and each dimension requires the appropriate unit - linear units for length, square units for area, and cubic units for volume (Lehrer, 2003; Stephan & Clements, 2003). Each understanding points to a different kind of student reasoning. A student who counts marks instead of intervals is attending to the wrong unit. A student who covers a rectangle with squares of different sizes has not established an equal unit. A student who expects a larger unit to produce a larger count has reversed the relationship between unit size and number of units. These understandings develop over time and follow identifiable developmental sequences across length, area, and volume (Battista, 2004; Barrett, Clements, & Sarama, 2017).

> **Key Idea • Measurement is one structure realized in several principles: a unit of the right kind, iterated without gaps or overlaps from an origin, counted, and conserved when the quantities are rearranged.**

Unitizing: Establishing, Iterating, and Composing Equal Units

Students establish an equal-sized unit, hold its size constant, and iterate it to compose a quantity. They coordinate the size of the unit with the number of units, and they can compose smaller units into a larger unit while preserving the quantity. A student who composes several smaller units into one new unit and then iterates that new unit is unitizing, and the same idea of composite units supports counting, place value, multiplicative reasoning, area, and fractions (Steffe, 1992; Reynolds & Wheatley, 1996; Lamon, 1996). When a student sees a quantity as composed of equal units rather than as separate pieces, the student no longer has to track every unit, which reduces demands on working memory (Sweller, 1988). Visual models can make this structure visible, especially when students first build it, then draw it, and finally represent it symbolically (Bruner, 1966; Paivio, 1986). Spatial and mathematical skills are distinct but related across development, with different spatial skills becoming more important at different ages (Mix et al., 2016).

> **Key Idea • Unitizing is establishing an equal-sized unit, holding its size constant, and iterating it to compose a quantity. Coordinating unit size with unit count is what gives a measurement its multiplicative structure.**

Measurement Magnitude: The Unit Is the Interval, Not the Mark

Length is the first attribute students formalize and the one whose difficulties are documented most precisely. Measurement magnitude is how much of the attribute is present. On a ruler, that magnitude is represented by the accumulated intervals from the starting point to the endpoint. The numeral at the endpoint names a location; it does not by itself name the length. On the 2003 National Assessment of Educational Progress, 86% of third graders, 78% of fourth graders, and 37% of eighth graders answered incorrectly when asked the length of an object that did not begin at the zero mark of a ruler (Blume, Galindo, & Walcott, 2007). In a series of experiments, kindergarten and second-grade children measured objects either aligned with the left edge of a ruler or shifted away from it, using either a conventional ruler or a row of discrete units. Children performed well on aligned problems and on shifted problems with discrete units, and they performed at chance on shifted problems with a ruler; adding numerals to the discrete units reduced their accuracy, and removing numerals from the ruler did not repair it (Solomon, Vasilyeva, Huttenlocher, & Levine, 2015). The obstacle is conceptual rather than procedural. Students read a ruler as a sequence of marks or numerals instead of a sequence of equal spatial intervals, so they count the wrong objects. Kamii and Clark (1997) reached the same conclusion from Piagetian assessments and argued that instruction reaches the tool before students have constructed the unit the tool represents. The misconception yields to instruction that confronts it: second graders randomly assigned to receive disconfirming evidence about their ruler strategy improved on shifted-ruler problems, while structural alignment alone did not produce the same effect (Kwon, Congdon, Ping, & Levine, 2024). A content analysis of three widely used elementary programs suggests why the misconception survives: they present measurement procedures before the conceptual principles those procedures depend on (Smith, Males, Dietiker, Lee, & Mosier, 2013).

> **Key Idea • The unit is the interval between the marks, not the mark, and the magnitude is the intervals accumulated from the starting point. A shifted object quickly reveals whether a student is reasoning about magnitude or reading the numeral at the end.**

CLASSROOM SNAPSHOT

A pencil rests beside a ruler. It begins at 3 and ends at 8.

Teacher: “How long is the pencil?”

Devin: “Eight. It stops on the eight.”

Maya: “It’s five. You have to start where it starts.”

Teacher: “Maya, how did you get five?”

Maya: “I counted the spaces. Three to four is one space, four to five is two. It’s five spaces.”

Teacher: “So one of you counted numbers, and one of you counted spaces. Which one is the pencil?”

Area and Spatial Structure: Organizing a Region Into Equal Units

Area requires students to organize a region into equal square units and the relationships among those units. Students see rows and columns before applying a formula, construct the complete array when the units are not all drawn, and decompose and recompose regions to reason from part to whole about a missing measure. Spatial structuring is how students organize what they see into units and relationships among those units (Battista et al., 1998). In two-dimensional arrays, many students count only the most visible squares, such as those around the border, because they have not yet organized the region into equal rows and columns (Battista et al., 1998). A formula cannot replace a structure the student has not built. The key insight concerns the relationship between the size of the unit and the dimensions of the rectangle it covers. Among 115 children in grades 1–4 who had received no formal area instruction, solution strategies fell into five levels, and progress depended on constructing a complete array of equal-sized units rather than drawing units of convenient size around the edges (Outhred & Mitchelmore, 2000). Students who have not built that array treat length times width as a rule about two numbers, and task-based work across grades 4–8 finds that most students do not treat the square as a unit that covers area even when they compute the product without error (Kamii & Kysh, 2006). The difficulty extends beyond children: in a teaching experiment with prospective elementary teachers, many could not explain why multiplying two linear measurements produces a measure of area, and had not constructed an image of a square unit from linear units (Simon & Blume, 1994). The array also carries multiplicative structure the student needs elsewhere. A student who sees a 6-by-8 rectangle as six rows of eight, as three groups of sixteen, or as (5 × 8) + (1 × 8) is composing and decomposing a quantity multiplicatively, and classroom studies of two-digit multiplication describe students' reasoning with rectangles partitioned into sub-rectangles in exactly this way (Izsák, 2004). The area model therefore represents the distributive property before it becomes an area formula. Curricular treatments of area show the same conceptual thinness documented for length (Smith, Males, & Gonulates, 2016).

> **Key Idea • Spatial structure is what a formula operates on. Students need to see area as rows and columns of equal square units before the formula has meaning, and the same array shows six rows of eight and (5 × 8) + (1 × 8).**

Relational Thinking: Coordinating Area and Perimeter

Area and perimeter are two different attributes of the same figure. Changing a figure can preserve one while changing the other, and students who fuse the two measures expect them to increase and decrease together. The intuition that figures with the same perimeter must enclose the same area is associated with how much geometry instruction students have received (Dembo, Levin, & Siegler, 1997). Three kinds of tasks separate the measures. Rectangles with the same area can have different perimeters. Rectangles with the same perimeter can enclose very different areas: a 6 by 6 square and an 11 by 1 rectangle both have a perimeter of 24 units and enclose 36 and 11 square units. Changing a figure's dimensions affects the two measures differently, which is where scaling later enters. In each case the question is the same: what remains invariant, and what changes?

> **Key Idea • Area and perimeter are distinct measures of the same figure. A student who expects them to move together holds one belief rather than two separate mistakes, and the remedy is to hold one measure constant while the other changes.**

Volume: Coordinating Three Dimensions at Once

Volume compounds the structuring demand because students must coordinate rows, columns, and layers while most units remain hidden. Clinical interviews with 123 third and fifth graders enumerating cubes in pictured three-dimensional arrays revealed a progression from counting the faces visible on the outside of the array, which double-counts corners and omits the interior, toward seeing the array as a set of congruent layers built from rows and columns (Battista & Clements, 1996). A classroom teaching experiment traced the same conceptual progress in fifth graders working through a sequence of cube-prediction tasks (Battista, 1999). That progression is a chain of composite units: five cubes compose a row, four rows compose a layer of twenty, and three layers compose a prism of sixty. A student who holds the chain tracks three groups of twenty rather than sixty separate cubes, which is the same unitizing that governs place value and fractions. Part of the difficulty concerns the representation rather than the solid, since students in grades 5–8 have marked trouble relating an isometric drawing to the rectangular solid it depicts (Ben-Haim, Lappan, & Houang, 1985). Whether children apply the row–column–layer structure or fall back on counting depends on task and material features, which puts the choice within reach of instruction (Vasilyeva et al., 2013).

> **Key Idea • Volume develops through composed units: cubes compose a row, rows compose a layer, and layers compose the prism. A student who sees three layers of 20 is unitizing rather than counting 60 separate cubes.**

Angle as Magnitude: Measuring an Amount of Turn

Angle is the attribute least anchored in a countable object, and students meet it in physical situations that look nothing alike: corners, turns, slopes, openings, and directions. An angle measures turn, not a judgment about how an angle looks. Students have to read the amount of turn rather than the figure's appearance, its orientation, or the length of its rays. In a cross-sectional study of 192 students in grades 2–8 modeling nine such situations, the standard angle concept appeared first where both arms of the angle are physically visible, and substantial proportions of students, even at grade 8, did not apply it to turning and sloping situations (Mitchelmore & White, 2000). The concept develops by progressive abstraction across situations rather than by definition. Observational work with fourth graders describes how the idea of turn as a body movement becomes coordinated with turn as a number, with full-body rotation gradually curtailed into smaller gestures and then into anticipation (Clements & Burns, 2000); those students were of above-average mathematical ability, so the sequence describes how the coordination can develop rather than a norm for typical classrooms.

> **Key Idea • An angle is an amount of turn. A student who judges an angle by its appearance, its orientation, or the length of its rays is not yet reasoning about magnitude.**

Composing and Decomposing Figures

The same compose-and-decompose action that operates on quantities operates on figures. Children progress from treating shapes as separate objects toward combining them deliberately and then recognizing the result as a new composite figure (Clements, Wilson, & Sarama, 2004). Asking what shapes compose a figure, how else it can be partitioned, and what remains invariant when the pieces are recomposed develops the same structuring that area and volume require.

> **Key Idea • Composing and decomposing figures is the same action students use on quantities, and it is the structuring that area and volume depend on.**

Measurement and Fractions Rest on the Same Unit of 1

Fractions apply the structure of the preceding sections to a unit of 1. Partitioning the unit of 1 into three equal units creates units of size one third, and two thirds is a count of two of those units, so the denominator names the size of the unit and the numerator names how many are iterated. An area model makes the partition visible only when students attend to the equivalence of the units; shading two regions of a rectangle demonstrates nothing about fractions if the regions are unequal. The strongest experimental evidence separates the work the two models do.

Second- and third-graders randomly assigned to number line training improved on an untrained fraction magnitude comparison task. In contrast, students assigned to area model training improved only on area model tasks (Hamdan & Gunderson, 2017). The area model shows partition and composition, and the number line represents magnitude, which argues for assigning the two representations different jobs rather than treating them as interchangeable.

> **Key Idea • A fraction is a count of equal units partitioned from the unit of 1. The area model shows the partition, and the number line represents the magnitude; the two are not interchangeable.**

What Geometric Measurement May Predict, and What the Research Does Not Yet Show

Spatial reasoning and number-line knowledge are associated with later mathematics achievement, and longitudinal studies show that early spatial skills can predict later mathematics performance (Atit et al., 2022; Gilligan, Flouri, & Farran, 2017; Gunderson et al., 2012; Schneider et al., 2018). Measurement itself has not been isolated as a predictor of later algebra. Still, geometric measurement develops understandings that matter across mathematics, and the four constructs name them: establishing and iterating an equal-sized unit and composing smaller units into larger ones (Unitizing); treating the interval rather than the mark as the unit and accumulating intervals from an origin (Measurement Magnitude); organizing a region into rows and columns of equal units (Spatial Structure); and coordinating area with perimeter to determine what is preserved and what changes (Relational Thinking). These developmental understandings are important to diagnose and develop across grades 2–8.

> Key Idea • Spatial and number-line competencies predict later mathematics. Measurement itself has not been isolated as a predictor, but these developmental understandings are important to diagnose and develop.

Building the Geometric Measurement Math Learning Profile: What the Diagnostic Should Reveal

A diagnostic built on these four constructs asks a different question of each one. A Unitizing item requires the student to identify, maintain, compose, or iterate equal-sized measurement units. A Measurement Magnitude item requires the student to reason about how much of the attribute is present rather than read a numeral, count marks, or apply a procedure. A Spatial Structure item requires the student to organize the figure into units and relationships among units rather than count visible pieces or recall a formula. A Relational Thinking item requires the student to coordinate two measures and determine what is preserved and what changes. The form carries 19 items at each grade, selected-response only, with one DOK 3 item per form as the Strategy Flexibility probe, and eighteen named misconception codes behind the wrong answers.

Read across the constructs, a pattern of responses locates the developmental understanding rather than the missed item. A student who establishes and iterates a unit correctly but misses the items that shift the origin can construct a unit and iterate it, and reads the tool as numerals rather than as intervals accumulated from a starting point; the unit is not the problem, the ruler is, and the next move is to measure something that starts at 2 and ask why the answer is not the number it ends on. A student who misses both coordination items has fused area and perimeter into one idea and expects the two to move together, which is usually one belief rather than two separate mistakes; the next move is to build every rectangle with an area of 24 square units and watch the perimeter change, then every rectangle with a perimeter of 24.

Strategy flexibility cuts across these developmental understandings. Students increasingly recognize structure and use it rather than counting every unit or applying a remembered procedure. In the MLP, Strategy Flexibility is reported separately because it describes how students use the underlying structure rather than naming another measurement construct.

> **Key Idea • The diagnostic identifies the developmental understanding underneath a response, not a list of measurement skills. A pattern of responses points to Unitizing, Measurement Magnitude, Spatial Structure, or Relational Thinking, and Strategy Flexibility is reported alongside them.**

Conclusion

Geometric measurement is more than learning to use a ruler or remember a formula. Students have to understand the unit being measured, how those units compose the quantity, and what the number represents. Common errors across grades 2–8 show where that understanding is still developing: reading ruler marks instead of intervals, covering an area without equal square units, counting only the visible cubes in a prism, or recognizing an angle in a corner but not in a turn. Short, well-chosen tasks can reveal these differences in reasoning. The order of instruction matters. Build the unit before introducing the tool. Build the array before introducing the formula. Build the layers before multiplying three dimensions. These same ways of reasoning support number lines, fractions, place value, multiplication, and proportional reasoning. Read through the Math Learning Profile; a student's response points to one of four developmental understandings: Unitizing, Measurement Magnitude, Spatial Structure, or Relational Thinking. The goal is not simply to help students get the measurement right. It is to help them understand what they are measuring and what they are counting.

> **Key Idea • The goal is to move students from reading a tool or applying a formula to understanding and counting the units that compose the quantity.**

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Social Media

You've seen it. A student counts 7 squares in a rectangle. You ask them to build the array. They write 12.

Nothing about their arithmetic changed. Everything about what they saw did.

A fourth grader is shown a rectangle with only the top row and left column of unit squares drawn—the rest blank—and asked how many squares will cover it. She counts what she can see and writes 7. Asked to build the array, she fills the region row by row and writes 12.

Nothing about her arithmetic changed. What changed is that she built the array.

Students who have built that structure:

- Count intervals, not marks, on a ruler
- Draw the complete array before multiplying
- See a rectangular prism as layers, not visible faces

This Insight examines how students come to count the units that fill a quantity—and what instruction can do when a tool or formula supplies the structure instead.

Inside, you'll learn:

- Why a ruler can conceal the very idea it was built to teach
- What the array shows about area that length × width does not
- How composing units into layers turns a count of cubes into a measure of volume
- Why the number line, read as a measurement model, is among the strongest correlates of later achievement

Read the full Insight: mathsuccess.io/dmtinsights

When your students hand you a correct measurement this week—do you know what they counted?