# DMT INSIGHTS

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## PATTERNS AND EQUALITY: FROM COMPUTING A TOTAL TO REASONING ABOUT A RELATIONSHIP

### A Correct Answer Can Conceal the Reasoning: Why This Construct Needs a Diagnostic

By grade 4, most students can complete 8 + 5 = ___ and continue 3, 6, 9, ___ without hesitation. Ask the same students to solve 8 + 4 = ___ + 5 and many write 12, while others write 17. The arithmetic is not the problem. The second item exposes how a student reads the equal sign: as an instruction to compute, or as a statement that two expressions name the same quantity. Most elementary students hold the first reading, and it survives long after they compute accurately (Falkner, Levi, & Carpenter, 1999).

Patterns and equality are paired in one construct because they develop together and because algebra asks for both. Equality supplies the idea that a quantity can be written in many forms without changing. Pattern work supplies the idea that a relationship can be described once and then applied to every case. A student who holds both can transform an expression and justify the transformation. A student who holds neither can compute correctly for years and still be unprepared for $3x + 4 = 19$.

The difficulty is that this reasoning is nearly invisible in ordinary classwork. Relational and operational thinkers answer standard computation items the same way, so a page of correct arithmetic does not distinguish them. How well students appear to understand the equal sign depends substantially on which equation structures a task uses (Stephens, Knuth, Blanton, Isler, Gardiner, & Marum, 2013). Making the difference visible takes items built for it, which this Insight and the Math Learning Profile (MLP) set out to supply.

> **Key Idea • Patterns and equality are the arithmetic roots of algebra, and correct computation does not reveal whether a student holds them. Assess the reasoning directly.**

## The Construct's Scope: What Belongs Here

Patterns and Equality develop relational reading of the equal sign, generalization of numeric and growing patterns, the properties of operations as a quantity-preserving structure, compensation within computation, and the first symbolic expression of generality. This construct asks students to reason about relationships among expressions and equations rather than compute. The Math Learning Profile organizes these ideas under three connected developmental understandings, described after the next section.

Neighboring constructs carry what is adjacent. Students develop the base-ten structure they compose and decompose in Place Value. Multiplicative comparisons of two quantities, ratio tables, and proportional reasoning belong to Scaling and Comparing. The unit of 1, partitioning, and equivalence among fractions belong to Fraction Concepts. Addition and Subtraction and Multiplication and Division develop the operations themselves, including the problem structures and the strategies students bring to them. This construct draws on all of them, because equality has to be about something, and it develops only the relational reasoning that runs across them.

> **Key Idea • Patterns and Equality develop the relational reasoning that applies across the operations. Place Value, Scaling and Comparing, Fraction Concepts, Addition and Subtraction, and Multiplication and Division develop the quantities themselves.**

## The Shift: The Object of Reasoning Moves from the Total to the Relationship

In arithmetic, the object of a student's reasoning is a total. The student reads 8 + 4, produces 12, and finishes the work. In algebra, the object of reasoning is a relationship between expressions, and the total may never be produced at all. Deciding that 37 + 48 = 40 + 45 requires no sum. It requires seeing that three was moved from one addend to the other, and that moving it changed nothing.

The shift has two halves. The first is *equivalence*: recognizing that two expressions written differently name the same quantity. Hence, an equation states that both expressions name the same quantity rather than marking a place to record a result. The second is *generalization*: recognizing that a relationship holding in several cases can be described once, in words, in a table, and eventually in symbols, and that the description then covers cases the student has not examined. Coordinating the two halves converts arithmetic into algebraic reasoning (Kaput, 2008).

Neither half arrives on its own. Grades 3–6 are when the shift can be built, and when its absence begins to cost. Middle school students who have not made it continue to read equations operationally well into grade 8, and their progress across three years is slow, uneven, and sometimes regressive (Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007). In contrast, students who receive sustained early-algebra instruction in grade 3 outperform their peers on equivalence, generalized arithmetic, and functional thinking, and use more algebraic strategies to get there (Blanton, Stephens, Knuth, Gardiner, Isler, & Kim, 2015).

> **Key Idea • The construct asks students to change what they reason about, from the total an expression produces to the relationship between expressions. Grades 3–6 are when that change can be built deliberately.**

## The Math Learning Profile: Three Connected Developmental Understandings

The MLP for Patterns and Equality is built on that shift. It organizes the construct into three connected developmental understandings that build in complexity: Equality as a Relationship, Patterns, and Algebraic Reasoning. Equality is the foundation, patterns generalize the relationship, and algebraic reasoning uses a known relationship to reach an unknown one. The mathematical ideas this Insight discusses (relational equality, generalization, the properties of operations, compensation, and symbolic generality) sit within or develop across these three understandings. They are not five separate subconstructs.

**Equality as a Relationship.** The equal sign means "the same as." Students coordinate the quantities on both sides of an equation, recognize that expressions written differently can name the same quantity, and reason with the unknown in different positions.

**Patterns.** Students recognize what changes and what stays the same, and describe how the quantities are related. The development runs from noticing what changes, to describing the relationship, to using the relationship to determine another term or a distant one, to generalizing it. Term-to-term thinking is where students begin, and a general relationship, represented in words, tables, models, and eventually symbols, is where they are headed.

**Algebraic Reasoning.** Here the term does not mean formal algebra or solving equations. It means using a known relationship to reason about an unknown one by tracking what changed, by how much, and what stayed the same. Students use the properties of operations as structure, compensate, compose and decompose quantities while preserving the relationship, reason from a known relationship or property rather than recomputing, and connect models and equations that represent the same structure. Symbolic generalization grows out of this reasoning in the later grades.

The sections that follow take up these ideas in order. Relational equality develops the first understanding. Generalizing patterns develops the second. The properties of operations, compensation, and symbolizing generality develop the third, and symbolizing also draws on the unknown from the first and the pattern rule from the second.

> **Key Idea • The MLP organizes Patterns and Equality into three connected developmental understandings: Equality as a Relationship, Patterns, and Algebraic Reasoning. Every idea in this Insight develops within one of them.**

## Relational Equality: The Equal Sign States a Relationship, Not a Command

A *relational understanding* reads the equal sign as "the same as." The expressions on either side name one quantity, whatever form each takes. The *operational understanding* that precedes it treats the symbol as an instruction to compute the left side and write the result on the right. Kieran (1981) documented the operational reading and its persistence into the later grades, and Carpenter, Franke, and Levi (2003) describe the relational alternative as attending to an equation as a whole structure rather than as a left-to-right procedure.

Nearly everything algebra does with an equation depends on the relational reading. A student who holds it can add the same quantity to both sides, rewrite one side, or recognize two equations as equivalent, because each move keeps both sides naming the same quantity, and the student can see why. Among middle school students, understanding of the equal sign is associated with success in solving algebraic equations, and the association holds after accounting for general mathematics ability (Knuth, Stephens, McNeil, & Alibali, 2006). The operational reading is not a deficit in capacity. It is the reasonable generalization of years of equations written a + b = c, and it resists instruction precisely because it has worked so well for so long (McNeil, 2014).

Number sentences in unfamiliar forms reveal whether the relational understanding is in place, especially when a true-or-false judgment can be settled without computing. Is 8 + 4 = 10 + 2 true? Find the missing number in 6 + 7 = ___ + 8. Is 3 = 3 a sentence, and what makes it true? Because the form of the equation determines much of what a task measures, a profile has to sample several forms rather than rely on one (Stephens et al., 2013). Across grades 3–6, the arc runs from judging whole-number sentences, to sentences carrying an operation on both sides, to sentences whose numbers are large enough that computing is unattractive, to sentences in which an unknown appears on either side.

> **Key Idea • A relational reading of the equal sign is the construct's foundation. Assess it with sentences written in varied forms, and listen for whether the student reasons about whether both sides name the same quantity rather than computes.**

**CLASSROOM SNAPSHOT**

*Mr. Alvarez has written 6 + 7 = ___ + 8 on the board.*

**Maya:** I got 13. Six and seven is thirteen.

**Mr. Alvarez:** Where does the thirteen go?

**Maya:** In the blank. That is the answer.

**Devin:** But then it says thirteen plus eight, and that is twenty-one. Those are not the same.

**Mr. Alvarez:** Devin, what number did you write?

**Devin:** Five. This side is thirteen, so that side has to be thirteen too. Eight is one more than seven, so I took one off the seven, and that leaves five.

**Maya:** Wait. So both sides have to be thirteen?

**Mr. Alvarez:** Is 6 + 7 = 13 + 8 true?

**Maya:** No. No, it is not.

## Generalizing Patterns: The Goal Is the Rule, Not the Next Term

*Generalization* describes the structure a pattern follows so the description covers every case, including cases the student has not seen. A student who extends 3, 6, 9, 12, 15 by adding three has continued the pattern. A student who says that each term is three more than the one before it, that each term is a multiple of three, and that the tenth term is thirty because ten threes are thirty, has described it.

This description later becomes a variable expression. Generalizing is one of the earliest forms of algebraic reasoning, and the entry point Kaput (2008) identifies for school algebra. Children in the elementary grades reach it earlier than curricula typically assume when instruction asks them for it (Carraher, Schliemann, Brizuela, & Earnest, 2006). The move also changes what students attend to. Eight-year-olds working with visual growth patterns can be brought to reason about the relationship between the term number and the term itself rather than only about the step from one term to the next, and it is that relationship, not the step, that generalizes (Warren & Cooper, 2008).

Items reveal generalization when they ask for a distant term and its justification, and when they ask whether a rule always holds. Here is a pattern: 3, 6, 9, 12. What is the tenth term, and how do you know? Describe the rule in your own words. Will it always work? A student who can produce only the next term is extending—a student who can predict a distant term without listing the terms before it is generalizing. Across grades 3–6, the arc runs from noticing what changes and what stays the same, to describing the relationship, to using it to predict a distant term, to generalizing the rule in words and in a table, and finally to writing it as an expression in which a letter stands for the term number.

> **Key Idea • Pattern work aims at a rule that covers every case. Assess it by asking for a distant term, for the rule in the student's own words, and for a justification that the rule always holds.**

## The Properties of Operations: Structure That Preserves Quantity

The commutative, associative, and distributive properties are ways to rewrite an expression without changing the quantity it represents. They open the third MLP understanding, Algebraic Reasoning: reasoning from a known relationship or property by tracking what changed and what stayed the same. A student uses the commutative property in recognizing that 4 + 9 and 9 + 4 name the same total. A student uses the associative property to solve 6 + 7 + 4 by composing 6 and 4 to make a complete unit of 10 and then adding 7. A student uses the distributive property to reason that 7 × 8 is 5 × 8 plus 2 × 8, having decomposed 7 into 5 and 2.

Each rewrite produces an equivalent expression, which makes the properties the place where equality stops being a claim about two given expressions and becomes a tool for producing new ones. In DMTI's structural language, these are acts of composing and decomposing quantities while holding equality fixed, and students should be able to narrate them: I decomposed seven into five and two; I composed six and four to make ten; I changed one quantity and adjusted the other so the total stayed equal. Understood this way, the properties organize what a student already knows into a connected structure rather than adding isolated rules (Hiebert & Carpenter, 1992). Learned as vocabulary, they do the opposite. A student can supply the word *commutative* and still verify 6 × 4 = 3 × 8 by computing both sides, which is instrumental understanding wearing a technical name (Skemp, 1976).

Items that block computation and ask for justification expose the structure. Without finding both products, is 6 × 4 = 3 × 8 true, and how do you know? How does knowing 5 × 8 help you find 7 × 8? Across grades 3–6, the arc runs from using a property implicitly inside a strategy, to describing the rewrite in words, to justifying an equality by the rewrite alone, to naming the property once the reasoning is secure.

> **Key Idea • The properties of operations are rewrites that preserve quantity. Assess them by asking students to justify an equality without computing, and listen for composing and decomposing language rather than for the property's name.**

## Compensation: Equality Made Visible Inside Computation

*Compensation* is changing a quantity in one place and adjusting in another so that a total, a difference, or a product stays the same. A student who solves 17 + 19 as 17 + 20 − 1 has raised one addend to a friendly number and then taken the extra back. A student who reasons 39 + 28 as 40 + 27 has moved one unit from the second addend to the first. The total never changes, because every move is matched.

Compensation is where relational equality appears in arithmetic students are already doing, so it is both the most accessible evidence of the construct and the most useful. It depends on benchmark reasoning, so it ties the construct to number sense: a student compensates toward a complete unit of 10 or 100 because those totals are easy to hold. It also generalizes. The same reasoning that holds a sum fixed holds a product fixed when one factor is doubled and the other halved, and holds a difference fixed when both numbers move the same way, as in 15 − 7 = 16 − 8.

Items reveal compensation when they ask a student to preserve a quantity rather than produce one. Is 37 + 48 = 40 + 45 true? Explain without adding. If I change 49 to 50, what must I do to 26 for the sum to stay the same? If I double one factor, what must happen to the other for the product to stay the same? The characteristic error is one-sided: the student adjusts one quantity and leaves the other alone. Across grades 3–6, the arc runs from compensating within addition, to compensating within subtraction, where both quantities move the same way, to compensating within multiplication, to stating the general rule for what the second quantity must do.

> **Key Idea • Compensation makes equality visible inside computation. Assess it by asking students**

**to hold a sum, a difference, or a product fixed and explain why the adjustment preserves it.**

## Symbolizing Generality: A Letter Names a Quantity, Not a Blank

The last idea in this construct is the move from describing a relationship in words or in a table to writing it with a symbol, together with recognizing that the symbol names a quantity. In 3 + n = 11, the letter n is a number, not a slot. In the statement that the tenth term is 3 × 10, the ten is one case of a rule that can be written 3 × n for any term number n. The unknown and the variable are two uses of the same idea, and grades 3–6 are where both begin.

Here the construct becomes recognizable as algebra, and here a shallow version fails. A student who treats a letter as a label, a blank, or an abbreviation for an object rather than as a quantity will later write expressions that cannot be evaluated or transformed. The reasoning that prevents this is built into the ideas above, because a symbol can stand for a quantity only if the student already accepts that a quantity can be written in many forms without changing. Instruction that moves from a model to a table to symbols supports the transition, and fading from concrete representations toward abstract notation is more effective than either representation used alone (Fyfe, McNeil, Son, & Goldstone, 2014).

Items reveal understanding when they ask what a symbol stands for and whether two symbolic statements say the same thing. If n + 5 = 12, what is n, and what does n mean here? A pattern's rule is 4 × n. What does n stand for? Are n + n and 2 × n the same? Across grades 3–6, the arc runs from a box for an unknown, to a letter for an unknown in a single equation, to a letter for a varying quantity in a pattern rule, to comparing two expressions written with the same letter.

**Key Idea • A letter names a quantity, and grades 3–6 are where the unknown and the variable begin. Assess the understanding by asking what the symbol stands for and whether two expressions say the same thing.**

## Reading the Errors: Each Mistake Names an Understanding to Rebuild

Errors in this construct are rarely careless. Each one identifies which understanding is missing, which is why they are worth collecting rather than only correcting. Writing 12 or 17 for 8 + 4 = ___ + 5 is the operational reading of the equal sign, and so is rejecting 3 = 3 or 8 + 6 = 7 + 7 as not real sentences because no answer follows the symbol. Extending a pattern correctly while being unable to state the rule is continuation without generalization, and it shows most clearly when a student asked for the tenth term begins listing. Treating every pattern as repeating, so that growing and functional patterns go unrecognized, is a narrower version of the same gap. Adjusting one quantity and not the other, as when 49 + 26 becomes 50 + 26, is a failure to coordinate what changed with what must stay the same, not a slip. Supplying a property's name while still verifying by computation is the label without the structure. Treating a letter as an abbreviation for an object produces expressions the student cannot use.

These errors are also predictable in their timing. Because years of successful arithmetic build the operational reading, it grows stronger before it weakens, and instruction that activates familiar arithmetic patterns can make equation performance worse rather than better (McNeil & Alibali, 2005). The implication is not to hold arithmetic back. It is to vary the forms in which equations appear early and often, so that no single form becomes a student's definition of what an equation looks like.

> **Key Idea • Each characteristic error names a specific missing understanding: the equal sign, the rule, the compensating change, the structure, or the symbol. The error tells a teacher what to rebuild, not merely that an answer was wrong.**

## Why Structure Is Easier to Hold: Load, Schema, and Transfer

Reasoning relationally is not only more faithful to the mathematics. It is less work. A student who decides that 37 + 48 = 40 + 45 by tracking a single adjustment holds far less in mind than a student who computes both sums and compares them, and reducing what must be held at once leaves attention available for the relationship itself (Sweller, 1988). This is why a relational reading often looks faster in a classroom, and why a student who holds it can handle larger numbers without apparent effort.

Relational understanding also produces schemas that organize what would otherwise be separate problems. Understanding depends on how knowledge is connected, and equality is unusually well placed to connect things (Hiebert & Carpenter, 1992). Once a student holds a schema for equality as a relationship, 14 = 9 + 5, 8 + 6 = 7 + 7, and $3x + 4 = 19$ are one idea in three forms. Building that schema depends on meeting the idea in more than one form. Learners who work through two analogous cases induce the shared structure and transfer it, while learners given a single case usually do not (Gick & Holyoak, 1983). That finding directly argues for varying equation forms deliberately, not incidentally.

The payoff is transfer, and transfer depends on what students notice. Experts sort problems by the principle that governs them, while novices sort by surface features (Chi, Feltovich, & Glaser, 1981). A student who reads the equal sign relationally is sorting by principle within arithmetic, years before the word algebra appears.

> **Key Idea • Relational reasoning lowers what a student must hold in mind, builds a schema that connects superficially different equations, and supports transfer. Schemas form from varied forms of one structure, not from repetition of one form.**

## The Math Learning Profile: What the Diagnostic Reveals and What It Points To

The Math Learning Profile for Patterns and Equality measures three connected developmental understandings, Equality as a Relationship, Patterns, and Algebraic Reasoning, not the arithmetic that carries them. The ideas above sit within those three. That distinction governs item design. Every item is built so that a student who computes accurately but reasons operationally answers differently from a student who reasons relationally, which usually means placing an operation on both sides, choosing numbers large enough that computing is unattractive, or asking for a justification instead of a value.

Item form is itself a measurement decision. Estimates of how well students understand the equal sign vary substantially with the equation structures a task uses, so a profile built on a single form will misreport (Stephens et al., 2013), and combining several item types onto one scale gives a far better picture than any one type gives alone (Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012). Understanding of equivalence is gradual rather than present or absent. A construct map built across grades 2–6 distinguishes levels that run from a rigid operational view, through a basic relational view, to comparative relational reasoning that operates on expressions without computing, and most students reach only the middle of that range by grade 5 (Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). The profile should report a level, not a verdict.

The point of the level is the next instructional move. A student at the operational level needs varied equation forms and true-or-false judgments before anything else. A student who judges sentences correctly but cannot explain them needs compensation items that force the adjustment into words. A student who changes one quantity but does not coordinate the related quantity needs a known fact with one change, and four questions: What changed? By how much? What has to happen to the related quantity? What stayed the same? A student who generalizes in words but not in symbols needs the bridge from a table to an expression, not more pattern extension. This kind of instruction works, and teachers can learn it. Professional development that focused teachers on eliciting children's algebraic reasoning produced measurable gains in students' understanding of the equal sign and in their relational thinking (Jacobs, Franke, Carpenter, Levi, & Battey, 2007), and attending to children's thinking as the basis for instructional decisions is the core of the cognitively guided approach on which DMTI builds (Carpenter, Fennema, & Franke, 1996).

> **Key Idea • The diagnostic reports a level in relational reasoning, not a score on arithmetic; it uses several equation structures because the form determines what the item measures, and each level names the instructional move that follows.**

## The Bridge: Equation Solving, Functions, and Proportional Reasoning Start Here

Patterns and equality are not an early-grade topic that grades 3–6 revisit. They mark where the object of a student's reasoning shifts from the total an expression produces to the relationship between expressions, and where a relationship is first described and then applied to every case. A student who can explain why 37 + 48 = 40 + 45 without adding, or why the tenth term of a pattern must be thirty without listing the first nine, is already doing the work that equation solving, functions, and proportional reasoning will require.

The instructional implication is specific. Vary the forms in which equations appear. Ask for a justification in place of a value often enough that students expect it. Make properties reasoning before they are vocabulary. Treat a characteristic error as information about which understanding to rebuild. Assessment that reports where a student stands in relational reasoning, not whether the arithmetic was correct, gives teachers in grades 3–6 something they can act on while there is still time.

> **Key Idea • Patterns and Equality are the bridge from arithmetic to algebra. Assessing relational reasoning directly and early makes the crossing teachable.**

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## Social Media

A fourth grader solves 8 + 5 without hesitation and continues 3, 6, 9, ___ without looking up. Then you write 8 + 4 = ___ + 5 and she puts 12 in the blank.

The arithmetic is fine. The second item exposes how she reads the equal sign: as an instruction to compute, not as a statement that both sides name the same quantity. Most elementary students read it the first way, and they keep reading it that way long after they compute accurately.

Our latest DMT Insight examines Patterns and Equality as a Math Learning Profile construct: the three connected developmental understandings, Equality as a Relationship, Patterns, and Algebraic Reasoning, that carry students from arithmetic into algebra in grades 3–6, and the items that reveal which ones a student holds.

- Equality is a relationship. A student who solves 6 + 7 = ___ + 8 by reasoning rather than adding is ready for the equations algebra will ask.
- Patterns are about the rule. Predicting the tenth term without listing the first nine is generalization. Continuing the pattern is not.

- The form of the equation decides what you measure. A student can look relational on one structure and operational on another, so a diagnostic has to sample several.
- Errors name the gap. Writing 12 for 8 + 4 = ___ + 5 tells you exactly which understanding to rebuild.

Read the full Insight at mathsuccess.io/dmtinsights.

Which is harder for your grade 3–6 students: reading the equal sign as “the same as,” or explaining why a pattern rule always works?

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