DMT INSIGHTS

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MATHEMATICAL THINKING IN THE AGE OF DIGITAL DEPENDENCE

Introduction

Students today have more access to mathematical support tools than at any point in history. Calculators complete computations instantly, adaptive software guides students step by step through procedures, and artificial intelligence systems generate explanations, hints, and complete solutions within seconds. Students often appear highly successful while using these supports. Teachers across grades report the same concerns: students struggle to retain what they have learned, transfer ideas to new contexts, explain relationships, and reason flexibly once supports are removed.

The issue is not technology itself. The deeper issue is cognitive outsourcing, which occurs when a tool performs the mental work that students still need to do for themselves. Lasting mathematical understanding develops through effortful cognitive activity: organizing relationships, constructing models, coordinating units, retrieving connected ideas, and reasoning across representations.

Horvath (2026) argues that classroom technology often prioritizes speed, convenience, and task completion over understanding and memory formation. That is a position rather than a demonstration, and it is not an argument against all technology. Adaptive practice that requires active recall and student-constructed responses differs from recognition-based scaffolding that bypasses cognitive effort. Mathematics is especially exposed, because mathematics is cumulative and hierarchical. Students cannot reason proportionally while multiplicative relationships remain fragile, cannot solve equations flexibly while equivalence and operations overload working memory, and cannot model relationships. At the same time, quantities and units carry no meaning.

The question for instruction is not whether technology belongs in a classroom. The question is which mathematical experiences build connected understanding, and what part a workbook, a pencil, and a student-drawn model play in building it.

Mathematical Learning Depends on Connected Structures

Mathematics instruction has often been framed as a tension between understanding and memorization. That framing is a false distinction. Understanding depends on students building connected structures of meaning over time.

Working memory is the limited-capacity system that holds and manipulates information during reasoning (Baddeley, 1992), and conventional problem solving can consume that capacity in ways that interfere with the schema acquisition that constitutes learning (Sweller, 1988). Mathematics places heavy demands on it, because students coordinate quantities, operations, symbols, units, and relationships at once. Among children followed from kindergarten through Grade 5, the visuospatial component of working memory predicted growth in mathematics specifically, and first-grade number-system knowledge predicted that growth over and above general ability (Geary, 2011).

Consider a student solving. $\frac{3}{4}x + 5 = 17$. The student must coordinate fraction magnitude, equivalence, inverse operations, and symbolic structure at once. A proportional bar model shows the structure of the equation before the symbols are manipulated. In the first bar, the full value of x is partitioned into four equal $\frac{1}{4}$-units, marked by small ticks above. The 17 spans the right three $\frac{1}{4}$-units of x together with the 5, which shows that $\frac{3}{4}x + 5 = 17$, and therefore that $\frac{3}{4}x$ equals 12.

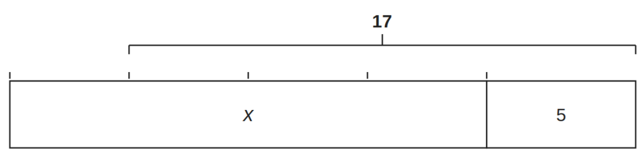

Figure 1. The full value of x is partitioned into four equal $\frac{1}{4}$-units. The 17 spans the right three $\frac{1}{4}$-units together with the 5.

Once students see that $\frac{3}{4}x = 12$, the next move follows. Three equal $\frac{1}{4}$-units of x make 12, so each $\frac{1}{4}$-unit equals 4. The second bar isolates the full value of x and brackets the same three $\frac{1}{4}$-units, now labeled 12. The fourth $\frac{1}{4}$-unit must also equal 4, so x equals 16.

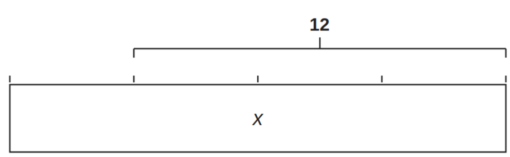

Figure 2. Isolating the full value of x. The same three $\frac{1}{4}$-units are now labeled 12, which reveals that each $\frac{1}{4}$-unit equals 4, and that x equals 16.

The progression asks students to reason structurally rather than to memorize isolated steps. They read one connected account of how parts compose a total rather than four separate procedural moves.

The point extends well beyond fact fluency. Mathematical reasoning depends on connected understanding of place-value relationships, fraction magnitude, multiplicative structures, proportional relationships, geometric composition, and symbolic equivalence.

Understanding develops recursively rather than by accumulation. Students return to important ideas, connect them to new ones, and reorganize the relationships among them through repeated opportunities to model, explain, compare, discuss, and generalize. They learn by organizing and reorganizing meaningful relationships over time rather than through exposure alone.

Consider a fifth grader meeting equivalent fractions. A student who has only watched examples on a screen may recognize that. $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent because the pairing is familiar. A student who has partitioned bars, folded strips, drawn number lines, and explained why both fractions name the same point has built a connected schema, and on meeting $\frac{3}{6}$, $\frac{4}{8}$, or $\frac{50}{100}$ recognizes the multiplicative structure rather than memorizing each case. Skemp (1976) named this relational understanding, knowing both what to do and why. Hiebert and Carpenter (1992) describe an idea as understood when its representation belongs to a network of connected representations.

> **Key Idea • Mathematical understanding develops when students build connected structures of meaning that support reasoning, transfer, and flexible problem solving.**

Why Representation Matters in Mathematics

Mathematics is abstract, but students do not learn abstraction directly. Understanding develops as students move from action and experience toward increasingly sophisticated representations.

Bruner (1966) described three modes of representation: enactive, or acting; iconic, or visual; and symbolic, or abstract notation. These are modes rather than fixed stages that every learner climbs for every topic. The distinction matters in mathematics because symbols alone often conceal structure, and students can manipulate them procedurally without understanding the relationships those symbols represent.

Mathematical models bridge that gap. Number lines, bar models, area models, and partitioned visual structures let students see relationships before expressing them symbolically. These models are not an incidental feature of a page. They are cognitive tools that reduce abstraction while preserving mathematical structure.

Students benefit most when they construct these models themselves rather than watching them appear. Neuroimaging studies comparing handwriting with typing find that forming letters and shapes by hand recruits motor, parietal, and language-related regions that keyboard entry does not, and improves later visual recognition of those forms (James & Engelhardt, 2012; Longcamp et al., 2008). Those studies examined letters and novel shapes rather than mathematical representations, so the extension to number lines and bar models is an inference rather than a finding. Drawing a number line or a proportional bar model asks a student to decide scale, partition space, coordinate quantities, and organize relationships visually.

Constructing a representation supports attention, strengthens encoding, and links motor activity with visual-spatial reasoning. Students organize relationships rather than select among answers. A flawed drawing is itself evidence. Uneven spacing and mislabeled partitions show a teacher what a student is thinking, and a tool that reports only a final answer conceals them. Dual coding theory proposes that the verbal and imaginal systems are separate yet interconnected (Paivio, 1986), so a student who coordinates symbolic notation with a drawn representation encodes the same relationship twice.

> **Key Idea • Constructing a representation by hand asks students to organize relationships that watching a finished one does not.**

Retrieval, Recognition, and Transfer

Retrieval strengthens learning across many subjects. Testing after study yields better long-term retention than restudying, and learners misjudge which of the two is more effective: immediately after practice, restudying feels more productive and yields higher confidence (Roediger & Karpicke, 2006). The evidence in mathematics specifically is weaker. A meta-analysis of spacing and retrieval practice for mathematics learning found a small effect for retrieval, with a confidence interval that crosses zero, alongside a more robust effect for spacing (Murray et al., 2025). The case for asking students to reconstruct rather than recognize, therefore, rests on the general finding and the structure of the mathematics, not on settled mathematics-specific evidence.

Many digital learning environments emphasize recognition rather than recall. Students identify correct answers from choices, follow guided prompts, or receive immediate scaffolding that reduces cognitive demand, thereby improving short-term performance. The evidence does not establish that a recognition format is inherently weaker than a recall format, so the question worth asking of a task is not its format but whether it asks the student to do the mathematical work the lesson intends.

A student who can complete a problem with step-by-step digital hints, but cannot complete a similar problem unaided the next day, has learned the script rather than the mathematics. Students need opportunities to explain relationships, reconstruct models, write equations, compare strategies, justify reasoning, and solve problems without immediate digital support.

A workbook that asks only for final answers in blank boxes is not the same as one that asks students to draw, label, and explain. Pages that ask for a drawn model, a written equation, and an explanation require students to reconstruct relationships rather than select among them, and varied practice across problem types asks them to compare relationships and recognize structures.

Transfer develops when students meet the same mathematical structure through multiple representations, contexts, and problem types over time.

> **Key Idea • Students build transferable understanding by reconstructing relationships across models, contexts, and problem structures rather than by recognizing them.**

Conclusion

Mathematics education, cognitive psychology, and neuroscience point in the same direction. Students learn mathematics most deeply when they construct relationships, organize ideas, draw models, retrieve connected understanding, and refine structures over time. Horvath (2026) makes the point most forcefully for younger learners, arguing that physically drawing, writing definitions, and constructing representations by hand builds structures that passive viewing does not. The primary evidence for that argument concerns letters and shapes rather than mathematical models, and its strongest claims outpace the studies. What the evidence does support is narrower and still useful. Forming a representation by hand engages motor, spatial, and language systems that a screen does not, and skipping that step in the name of efficiency carries a cost that does not appear immediately.

Technology does not have to be abandoned. It does have to stop performing the cognitive work that students still need to do for themselves. Students need opportunities to draw and model, compose and decompose units, partition and iterate quantities, compare strategies, justify reasoning, and explain structure.

A student who has built multiplicative reasoning through bar models and ratio tables in Grades 4 and 5 can apply the same structural thinking to proportional reasoning in Grade 6, slope in Grade 7, and linear functions in Grade 8. The surface details change, from sharing cookies to mixing paint to comparing speeds, while the multiplicative structure holds. Students who recognize that structure transfer their reasoning, and students who memorize procedures begin each year again.

A workbook, a pencil, and student-drawn number lines and bar models are not old-fashioned. They are the difference between a student who recognizes an answer and a student who can rebuild it.

The goal of mathematics instruction is not efficient task completion. The goal is students who recognize structure, transfer understanding, coordinate relationships, and reason flexibly across contexts.

> **Key Idea • Students learn mathematics by constructing, organizing, retrieving, and refining connected ideas over time.**

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Social Media

Students appear successful on a screen. Why do they struggle the moment the supports come off?

Because clicking through hints is not the same as building understanding.

This Insight examines why student-drawn models, handwritten work, and effortful retrieval matter for durable mathematical learning.

- **Cognitive outsourcing is the risk.** A tool that selects the operation, animates the model, and scaffolds every step leaves students recognizing answers rather than reasoning about them.
- **Constructing a model is not the same as watching one.** Forming a representation by hand engages motor, spatial, and language systems that a screen does not.
- **Reconstruction beats recognition.** A workbook that asks students to draw, label, and explain asks for the mathematics. One that asks only for a final answer asks for the answer.
- **Technology is not the problem.** Adaptive practice that requires active recall differs from step-by-step scaffolding that bypasses the need for effort. The question is whether the tool asks the student to do the work.

The point is not a return to an older classroom. The point is that drawing, modeling, retrieving, and explaining build understanding that a screen does not supply.

Read the full Insight at mathsuccess.io/dmtinsights.

Where do your students rely most on digital scaffolds, and what do they struggle to do without them?

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