DMT INSIGHTS

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Math Success by DMTI

MATHEMATICAL THINKING IN THE AGE OF DIGITAL DEPENDENCE

Introduction: Mathematical Understanding in a Digital World

Students today have more access to mathematical support tools than at any point in history. Calculators solve computations instantly. Adaptive software guides students step by step through procedures. Artificial intelligence systems generate explanations, hints, and complete solutions within seconds. In many classrooms, students appear highly successful while using these supports. Yet teachers across all grades continue to report the same concerns: students struggle to retain learning, transfer ideas to new contexts, explain relationships, or reason flexibly when supports are removed.

The issue is not simply technology itself. The deeper issue is **cognitive outsourcing** — when tools begin performing the very mental work students still need to develop internally. Recent work in cognitive psychology, neuroscience, and mathematics education suggests that lasting mathematical understanding develops through effortful cognitive activity: organizing relationships, constructing models, coordinating units, retrieving connected ideas, and reasoning across multiple representations.

Horvath's recent work in *The Digital Delusion* (Horvath, 2024) argues that modern digital environments often prioritize speed, convenience, and task completion over understanding and memory formation. This is not an argument against all technology. Adaptive practice that requires active recall and student-constructed responses is different from passive, recognition-based, or step-by-step scaffolding that bypasses cognitive effort. Mathematics may be especially vulnerable because mathematics is cumulative and hierarchical. Students cannot reason proportionally if multiplicative relationships remain fragile. They cannot solve equations flexibly if equivalence and operations overload working memory. They cannot model relationships if quantities and units are not meaningful.

For teachers, the central question is not whether technology should exist in your classroom. The question is: *What kinds of mathematical experiences actually help students build connected mathematical understanding — and how can a workbook, pencil, and student-drawn models play a non-negotiable role in that process?*

Mathematical Learning Depends on Connected Structures

For many years, mathematics instruction was often framed as a tension between "understanding" and "memorization." Cognitive science research now suggests this is a false distinction. Understanding itself depends heavily on students building connected structures of meaning over time.

Working memory refers to the limited amount of information the brain can actively process at one time (Baddeley, 1992). Cognitive Load Theory suggests that when too many elements must be coordinated simultaneously, students experience overload and performance declines (Sweller, 1988). Mathematics places particularly heavy demands on working memory because students must coordinate quantities, operations, symbols, units, and relationships simultaneously.

Consider a student solving the equation $\frac{3}{4}x + 5 = 17$. A student must coordinate the magnitudes of fractions, fraction equivalence, inverse operations, symbolic structure, and procedural reasoning simultaneously. A proportional bar model can help students see the structure of the equation before manipulating symbols procedurally. In the first bar, the full value of x is partitioned into four equal ¼-units, marked by small ticks above. The 17 spans the right three ¼-units of x together with the 5, showing that $\frac{3}{4}x + 5 = 17$ — and therefore that $\frac{3}{4}x$ must equal 12.

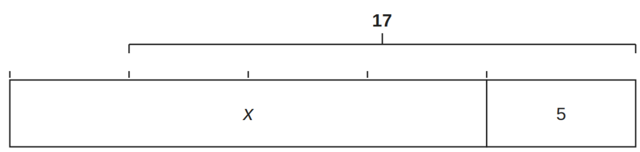

Figure 1. The full value of x is partitioned into four equal ¼-units. The 17 spans the right three ¼-units together with the 5.

Once students see that $\frac{3}{4}x = 12$, the next move follows naturally: $\frac{3}{4}x$ represents three equal ¼-units of x, so each ¼-unit must equal 4. The second bar isolates the full value of x and brackets the same three ¼-units, this time labeled 12. From there, students can see that the fourth ¼-unit must also equal 4, so the whole x equals 16.

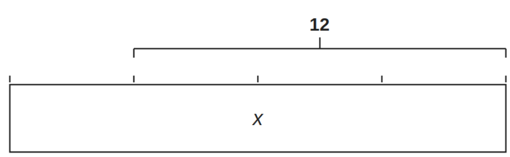

Figure 2. Isolating the full value of x. The same three ¼-units are now labeled 12, revealing that each ¼-unit must equal 4, and the whole x must equal 16.

This type of progression helps students reason structurally rather than memorize isolated steps. Students see that $\frac{3}{4}x + 5 = 17$ leads to $\frac{3}{4}x = 12$, that $\frac{1}{4}x = 4$, and that $x = 16$ — not as four separate procedural moves, but as a single connected story about how parts compose a whole.

This issue extends far beyond basic fact fluency. Successful mathematical reasoning depends on students building connected understanding of place-value relationships, fraction magnitude, multiplicative structures, proportional relationships, geometric composition, and symbolic equivalence.

Mathematical understanding develops more like weaving than stacking. Students do not simply place one skill on top of another; they return to important ideas, connect new strands, tighten relationships, and gradually create a stronger structure. Learning is recursive. Concepts are revisited and reorganized through repeated opportunities to model, explain, compare, discuss, and generalize.

Students do not learn mathematics deeply through exposure alone. They learn by organizing and reorganizing meaningful relationships over time.

Consider what happens when a fifth grader first encounters equivalent fractions. A student who has only watched examples on a screen may recognize that ½ and 2/4 are equivalent because they have seen the pairing many times. However, a student who has partitioned bars, folded strips of paper, drawn number lines, and explained why both fractions name the same point on a number line has built a connected schema. When that student later meets 3/6, 4/8, or 50/100, they do not need to memorize each case — they recognize the underlying multiplicative structure. This is why your students forget by Friday if they only watched on a screen. Cognitive psychologists describe this as the development of *relational understanding* — the ability to see how ideas connect rather than treating each procedure as an isolated rule (Hiebert & Carpenter, 1992; Skemp, 1976).

> Key Idea • **Mathematical understanding develops when students build connected structures of meaning that support reasoning, transfer, and flexible problem solving.**

Why Representation Matters in Mathematics

Mathematics is abstract, but students do not learn abstraction directly. Understanding develops gradually as students move from action and experience toward increasingly sophisticated representations.

Jerome Bruner (1966) described learning as progressing through three forms of representation: *enactive* (acting), *iconic* (visual representation), and *symbolic* (abstract notation). This progression is especially important in mathematics because symbols alone often conceal structure. Students may manipulate symbols procedurally without understanding the relationships those symbols represent.

Math models help bridge this gap. Number lines, bar models, area models, and partitioned visual structures allow students to see mathematical relationships before expressing them symbolically. These models are not decorative supports. They are cognitive tools that reduce abstraction while preserving mathematical structure.

Here is what the research says for your teaching: Students benefit most when they **physically construct these models**

themselves — not when they watch you draw them, not when a screen animates them. Research comparing handwriting and drawing with typing suggests that physically constructing representations activates broader neural systems involving language, motor processing, spatial reasoning, attention, and memory encoding (Horvath, 2024). Drawing number lines and proportional bar models requires students to decide scale, partition space, coordinate quantities proportionally, and organize relationships visually.

This physical construction process supports attention, strengthens encoding, links motor activity with visual-spatial reasoning, and reduces passive recognition. Students actively organize relationships rather than simply selecting answers on a screen.

When a student draws a flawed number line — uneven spacing, mislabeled partitions — that mistake is a gift. You can see their thinking. You can intervene. Digital tools often hide these errors behind a correct "final answer."

Students who never internalize multiplicative relationships, the magnitude of equivalent fractions, or symbolic structure often face major difficulties later, even if earlier digital performance appeared successful. This is one reason students need to draw number lines and proportional bar models themselves; the act of constructing these models supports reasoning, helps students coordinate quantities, and builds structural understanding.

Paivio's Dual Coding Theory (1990) further suggests that learning is strengthened when information is represented both verbally and visually. In mathematics, students who coordinate symbolic notation with drawing representations build richer and more connected mathematical understanding over time.

> Key Idea • **Drawing representations and math models help students construct, organize, and internalize mathematical relationships.**

Retrieval, Recall, and the Development of Transfer

One of the strongest findings in cognitive psychology is that retrieval strengthens learning. Students do not build mathematical understanding simply by seeing information repeatedly. Learning strengthens when students actively retrieve relationships, reconstruct ideas, and apply knowledge across contexts.

Many digital learning environments emphasize recognition rather than recall. Students identify correct answers from choices, follow guided prompts, or receive immediate scaffolding that reduces cognitive demand. These supports can improve short-term performance while simultaneously reducing the retrieval effort necessary for long-term understanding.

Here is the hard truth for teachers: If a student can solve a problem with step-by-step digital hints but cannot solve a similar problem on paper the next day, **they have not learned mathematics. They have learned to follow a script.**

Students need opportunities to explain relationships, reconstruct math models, write equations, compare strategies, justify reasoning, and solve problems without immediate digital support. Repeated retrieval strengthens the organization of mathematical ideas and supports transfer to new situations.

This is where a well-designed workbook matters. A workbook that only asks for final answers in blank boxes is not the same as one that requires students to draw, label, and explain. Avoid pages that reduce mathematics to fill-in-the-blank recognition. Varied practice worksheets — the kind where students draw, label, write equations, and explain — are not simply procedural repetition. They help students compare relationships, recognize underlying structures, and coordinate ideas flexibly across multiple situations. They require retrieval. They do not offer a bank of answers to recognize.

Transfer develops when students encounter the same mathematical structure through multiple representations, contexts, and problem types over time.

> **Key Idea • Transfer develops when students repeatedly retrieve and apply mathematical relationships across models, contexts, and problem structures.**

Conclusion: Building Mathematical Thinkers

Research from mathematics education, cognitive psychology, and neuroscience consistently points toward the same conclusion: students learn mathematics most deeply when they actively construct relationships, organize ideas, draw models, retrieve connected understanding, and refine structures over time.

Horvath (2024) makes this point especially forcefully for younger learners. When students are young, the act of physically drawing, writing down definitions, and constructing representations by hand helps build neural structures that do not form the same way through passive viewing. I've watched a sixth grader erase a bar model four times because the scale was wrong — and that struggle taught more than any hint button ever could. The motor act of writing — the deliberate shaping of letters, numbers, and diagrams — engages systems in the developing brain that support attention, memory, and meaning. In other words, when a child writes a definition or draws a bar model, they are *helping to build* the mental architecture that will hold that idea later. Skipping this step in the name of efficiency comes at a cost that does not show up immediately but accumulates over the years.

As a teacher, you do not need to abandon technology. You do need to stop letting it do the cognitive work your students still need to do themselves.

Students need opportunities to draw and model, compose and decompose units, partition and iterate quantities, compare strategies, justify reasoning, and explain structure. These experiences help students build understanding that is transferable, flexible, and usable in unfamiliar situations.

For example, a student who has built a strong understanding of multiplicative reasoning through bar models and ratio tables in fourth and fifth grade should be able to apply that same structural thinking when they meet proportional reasoning in sixth grade, slope in seventh grade, and linear functions in eighth grade. The surface details change — the context shifts from sharing cookies to mixing paint to comparing speeds — but the underlying multiplicative structure remains the same. Students who recognize that structure transfer their reasoning. Students who only memorize procedures start over each year.

Your workbook, pencil, and student-drawn number lines **and bar models** are not old-fashioned. They are evidence-based. They are the difference between a student who recognizes an answer and a student who truly understands.

Ultimately, the goal of mathematics instruction is not simply efficient task completion. The goal is to develop students who can think mathematically — students who can recognize structure, transfer understanding, coordinate relationships, and reason flexibly across contexts.

> Key Idea • **Students learn mathematics by constructing, organizing, retrieving, and refining connected ideas over time. A pencil and a blank page are still two of the most powerful tools in your classroom.**

References

Baddeley, A. (1992). Working memory. *Science, 255*(5044), 556–559.

Bruner, J. S. (1966). *Toward a theory of instruction*. Harvard University Press.

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A five-year longitudinal study. *Developmental Psychology, 47*(6), 1539–1552.

Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). Macmillan.

Horvath, J. C. (2024). *The digital delusion: How modern technology is reshaping the way we think and learn*.

Paivio, A. (1990). *Mental representations: A dual coding approach*. Oxford University Press.

Roediger, H. L., & Karpicke, J. D. (2006). Test-enhanced learning: Taking memory tests improves long-term retention. *Psychological Science, 17*(3), 249–255.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20–26.

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science, 12*(2), 257–285.

Social Media Post

Your students appear successful on a screen. So why do they struggle the moment the supports come off?

Because clicking through hints is not the same as building understanding.

Our latest **DMT Insight on Digital Dependence** looks at why student-drawn models, handwritten work, and effortful retrieval are not relics of an older classroom — they are the cognitive engine of durable mathematical learning:

- **Cognitive outsourcing is the real risk.** Tools that perform the mental work — selecting operations, animating models, scaffolding every step — leave students recognizing answers rather than reasoning about them.
- **Drawing builds understanding that watching cannot.** Physically constructing number lines and bar models engages motor, spatial, and language systems that screen-watching does not.
- **Retrieval beats recognition.** Workbooks that ask students to draw, label, and explain build transfer of knowledge. Workbooks that only ask for final answers build false confidence.
- **Technology is not the enemy.** Adaptive practice that requires active recall is different from step-by-step scaffolding that bypasses cognitive effort. The question is what the tool asks the student to do.

This is not about going backward. It is about giving students experiences — drawing, modeling, retrieving, explaining — that build the connected understanding no screen can provide.

Read the full research overview at **www.dmtinstitute.com** · **MathSuccess.io**

Where do your students rely most on digital scaffolds — and what do they struggle to do without them? Share your thoughts at contact@dmtinstitute.com.

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