DMT INSIGHTS

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WHY STUDENTS STRUGGLE WITH WORD PROBLEMS AND HOW BAR MODELS HELP

Introduction

Students who compute fluently often stall the moment a problem appears in words. They can add, subtract, multiply, and divide, but when those operations are embedded in language, the task becomes much harder. The arithmetic is rarely the obstacle. Students lack a way to organize the quantities and the relationships in front of them, and instruction that trains them to scan for keywords hands them a substitute for that organization rather than the organization itself. A student who searches for a trigger word guesses. A student who asks what the quantities are and how they are related reasons.

Bar models, also called strip diagrams or tape diagrams, represent quantities as lengths. A length is spatial, visible, and comparable, so students can see that one quantity is part of another, that two quantities compose a total, or that one exceeds another by some amount. The questions change with the representation. Instead of asking which word signals addition, students ask what the total is, what the parts are, what is known, and what is missing. Ng and Lee (2009) describe the reasoning the model method requires in three phases: reading the text, building the structure, and only then working symbolically.

> **Key Idea • Students who struggle with word problems usually are not failing at computation. They are failing to see the structure of the situation.**

Historical Foundations: The Singapore Model Method

The model method was developed in Singapore under the Primary Mathematics Project, which ran from 1980 to 1987, and it was introduced into primary textbooks in 1983 to represent the relationships between quantities in word problems (Kaur, 2019). It gave students a single consistent representation to build on before they formalized their thinking with equations.

Bruner (1966) describes a progression among modes of representation, from enactive experience through iconic representation to symbolic abstraction. A bar model belongs to the iconic mode. It is no longer the physical act of moving objects, nor yet a symbolic sentence. That position explains why the representation extends coherently across addition, subtraction, multiplication, division, fractions, ratios, and early algebra. Bruner does not discuss bar models, and placing them in the iconic mode is a reading of his framework rather than a claim he makes.

The model method was never a one-time strategy for a single problem type. Its strength is coherence: the same representational logic serves many topics and many grades. A student who uses bars to represent part–whole relationships in Grade 1 later uses the same structure to compare quantities, represent multiplicative relationships, and reason about unknowns in pre-algebra.

> **Key Idea • A bar model belongs to Bruner's iconic mode, between concrete experience and symbolic abstraction, and the same representation extends coherently from arithmetic through early algebra.**

What Bar Models Reveal About Mathematics

Bar models make visible the structure that symbols alone leave hidden. In join, separate, and part–whole situations, one bar represents a total that can be decomposed into parts. Students see quantities as connected components of a total rather than as isolated values.

Consider 245 = 200 + 40 + 5. The bar below shows how a three-digit number is composed of its place-value parts. Each rectangle is sized in proportion to its value, so the model itself carries what the digits mean: 200 is much larger than 40, which is much larger than 5.

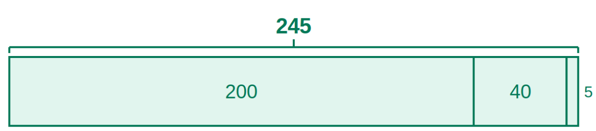

Figure 1. Place-value decomposition of 245. The bracket above the bar represents the total, and the parts are sized in proportion to their values.

In a comparison situation, two bars represent two quantities, and the difference appears as extra length. Students see the difference as a relationship rather than inferring it from words. Below, a 10-cm stick is compared with an 8-cm worm. The arrow between the bars marks the unknown: how much longer is the stick than the worm? Students see the relationship before they compute it, and then reason that the missing length must be 2 cm to complete the comparison.

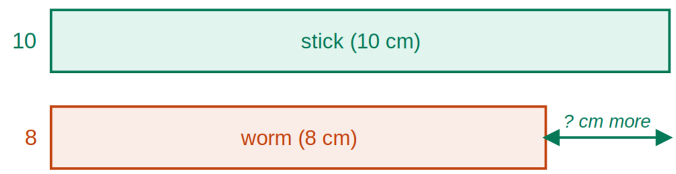

Figure 2. Comparison model. The two bars are drawn to scale, and the arrow marks the unknown difference in length between them.

When units are equal, these same structures extend into multiplication and division. A bar partitioned into equal units shows that a total can be composed through iteration. In every case, students begin with structure rather than with an operation, reasoning about how units compose and decompose rather than selecting a procedure from a list (Carpenter et al., 1999). What students draw matters here. Schematic representations, which encode the relationships in a problem, are associated with success in problem solving, while pictorial representations of the objects themselves are associated with failure (Hegarty & Kozhevnikov, 1999).

> **Key Idea • Most arithmetic and early algebra problems reduce to two structures, part–whole and comparison, and a bar model makes both visible.**

Cognitive Foundations: Why Bar Models Work

A bar model moves the relationship out of the student's head and onto the page, where it can be examined rather than held in the mind. External representations of this kind carry part of the reasoning themselves, because the arrangement on the page does work that would otherwise have to be done in memory (Zhang, 1997). That matters because a word problem asks students to process language, track quantities, and coordinate relationships at once. A heavy problem-solving load competes with the schema formation on which learning depends (Sweller, 1988).

Bar models also engage two codes at once. Dual coding theory proposes that the verbal and imaginal systems are separate yet interconnected (Paivio, 1986). That instruction pairing words with a relevant visual representation supports understanding more than words alone (Mayer, 2009). Over time, students recognize recurring structures, part–whole, comparison, and equal groups, and those structures become schemas. Schema-based instruction produces large gains for elementary students in both problem solving and transfer to unfamiliar problems (Peltier & Vannest, 2017).

A third mechanism is spatial. Quantities become lengths, and relationships become spatially visible. Spatial skill is moderately associated with mathematical performance across a large body of studies (Atit et al., 2022). The reasoning strategy that matters most here is identifying the value of one unit and then iterating or partitioning it, which connects multiplication, division, fractions, and ratios into one coherent way of thinking.

> **Key Idea • A bar model puts the relationship on the page, pairs a visual code with a verbal one, and builds schemas that carry to new problems.**

From Bar Models to Algebraic Thinking

Bar models support the transition from arithmetic to algebra. Students first encounter an unknown as a quantity within a structure rather than as an abstract symbol, and the representation that helped them compose parts and compare quantities now holds the unknown in place.

The bar below represents $4x + 5 = 53$, where x is an unknown quantity that appears four times. The four equal bars labeled x show the unknown iterated four times; the smaller bar of 5 completes the total, and the bracket above marks 53. A student who reasons that the four x bars together must equal 48, and therefore that each x must be 12, is doing the work that solving $4x + 5 = 53$ makes formal. The structure of the equation is present in the model before anyone writes it symbolically.

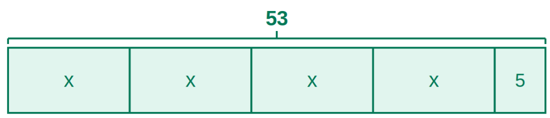

Figure 3. Algebraic reasoning. Four equal bars of x and a bar of 5 compose a total of 53, and reasoning about that structure leads directly to 4x + 5 = 53.

Bar models support relational thinking, which underlies algebra (Carpenter, Franke, & Levi, 2003). Students learn to read an equation as a statement about a relationship rather than as a procedure to execute, and that reading carries well past elementary school.

> **Key Idea • A bar model holds the structure of an equation before the equation is written, which gives students a meaningful entry into algebraic reasoning.**

Developmental Progression: How Bar Models Grow with Students

The representation evolves with the mathematics rather than simply repeating across grades. In the early grades, students use bar models for joining, separating, and part–whole situations, identifying the total and its parts. The same model then carries comparison situations and equal groups, where students coordinate several quantities and name relationships such as more than, less than, and times as many.

In the upper grades, the representation becomes more precise. Students use bar models to reason about fractions, scaling, and multiplicative relationships, partitioning units and iterating them to build new quantities. By middle school, the same structure holds unknowns, and the relationships that students draw lead directly to equations.

Students refine one representation rather than learning a new strategy each year. Among Primary 5 children, the model method provided a way to represent and solve algebraic word problems without access to letter-symbolic algebra, and their errors were usually a single misdrawn element rather than a failure of the whole approach (Ng & Lee, 2009).

> **Key Idea • One representation grows with the mathematics across grades K–8, carrying whole numbers,**

fractions, ratios, and early algebra. Students refine one tool rather than collecting many.

Using Bar Models Well: Where Instruction Goes Wrong

Bar models work as a tool for making sense of relationships, not as a set of drawing steps. When instruction focuses on reproducing a diagram, students produce correct-looking models without understanding the quantities they represent. The questions that matter are what a part represents, how the quantities relate, and where the unknown is.

Attention to units is the critical instructional focus. Every bar represents a quantity and, more importantly, a unit that students can compose, decompose, partition, or iterate. As problems grow more complex, students have to track what the numbers represent, not just the numbers themselves. Without that attention, a student can draw an accurate diagram that supports no correct reasoning.

Three failure modes recur. In the first, the bar model becomes a drawing procedure rather than a reasoning tool, and the diagrams are neat but do not reflect the quantities in the problem. In the second, unit reasoning is skipped, and the model becomes an incidental feature of the page rather than an analytical one. In the third, the bar model is reserved for difficult problems rather than used consistently, so students never build the habit of reasoning about relationships.

Key Idea • A bar model is effective in proportion to the attention students give to units and relationships, not to how accurately the diagram is drawn.

Conclusion

One representation, a length standing for a quantity, carries students from kindergarten part–whole problems into early algebra. Along the way, it makes structure visible, moves relationships from working memory onto the page, and gives students a stable anchor for reasoning about units.

Mathematics education and cognitive psychology converge here. Bar models make structure perceptible, support schema development, and promote transfer. Taught with an emphasis on units, relationships, and reasoning rather than on drawing rules, they build durable understanding from the early grades into algebra.

Key Idea • One coherent representation used across years builds deeper understanding than many disconnected strategies layered one on another.

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Social Media

Your students can compute. Why do word problems still stop them?

Because solving for an answer is not the same as understanding a relationship, a student who scans for a keyword produces an operation. A student who draws the quantities produces a structure, and the structure shows which operation fits.

This Insight examines why one representation, a length standing for a quantity, carries students from kindergarten part–whole problems into early algebra.

- Structure over keywords. Bar models show part–whole and comparison relationships, rather than leaving students to hunt for trigger words.

- Less load, more reasoning. Putting the relationship on the page frees working memory for thinking.
- One model, many years. The diagram a first grader uses for part–whole becomes the ground for fractions, ratios, and algebra.
- Units are everything. A model is only as strong as the student's understanding of what each bar represents.

The point is not a new strategy. The point is one consistent way to see structure, from kindergarten through algebra.

Read the full Insight at mathsuccess.io/dmtinsights.

Where do your students struggle most with word problems: seeing the structure or choosing the operation?