DMT INSIGHTS

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Math Success by DMTI

RATIO TABLES: BUILDING MULTIPLICATIVE AND PROPORTIONAL REASONING

What a ratio table records about units, scaling, and equivalence

Introduction

Students may recall multiplication facts accurately and still struggle to reason multiplicatively in unfamiliar situations. Fact knowledge and multiplicative reasoning are related, but they are not the same. Multiplicative reasoning requires students to coordinate quantities, identify units, relationships, and recognize when two quantities change proportionally. In a cohort of British children followed for five years, mathematical reasoning, which included multiplicative and proportional reasoning, predicted later mathematics achievement more strongly than arithmetic skill, with age, general ability, and working memory controlled (Nunes et al., 2012).

Ratio tables can make these relationships visible by organizing equivalent relationships/ratios within a common representation. The important mathematics is not the table itself. It is the relationships the table records.

This DMT Insight reviews the research on multiplicative reasoning, unitizing, representation, and transfer, and what that research suggests about ratio tables. DMT Companion 47 addresses instruction and coaching.

> **Key Idea • The important mathematics is not the table itself. It is the relationships the table records.**

From Additive to Multiplicative Reasoning

Additive reasoning asks how much more. Multiplicative reasoning asks how many times as much or by what factor (Thompson & Saldanha, 2003). Vergnaud (1988) described multiplication, division, fractions, ratio, and proportion as one part of the larger field of situations, the multiplicative structures, that students come to understand over many years. Reviews of the research describe the development of proportional reasoning as long and uneven (Lamon, 2007; Tourniaire & Pulos, 1985).

Additive reasoning is not inherently incorrect. It is a conception that works across many earlier situations. Difficulty occurs when students apply that additive relationship to a situation in which the quantities are related multiplicatively. Suppose 1 bag holds 4 apples. A student who records 6 apples for 2 bags and 8 apples for 3 bags has added 2 each time. The numbers follow a pattern, but the number of apples is no longer four times the number of bags.

Van Dooren et al. (2010) gave additive and proportional word problems to 325 students in Grades 3 to 6. Additive answers dominated in Grade 3, and proportional answers dominated in Grade 6. Nearly half of the fifth- and sixth-graders made both kinds of errors, applying each kind of reasoning where the other was needed. About 5 percent of the students answered all four problem types correctly. Students' choices depended more on whether the numbers formed whole-number ratios than on the problem type. A student's preference for additive or multiplicative relationships is also separable from the student's calculation skill (Degrande et al., 2018).

> **Key Idea • Additive reasoning is not inherently incorrect. Difficulty occurs when it is applied to multiplicatively related quantities.**

Unitizing and Composite Units

Multiplicative reasoning depends on unitizing: treating a collection as one unit that can then be iterated, partitioned, composed, or scaled (Lamon, 2007; Lobato & Ellis, 2010). Steffe (1992) described this as coordinating units. A student reasoning about three fours coordinates the unit of four with the number of times that unit is iterated.

In a situation in which 1 bag contains 4 apples, four apples can be treated as one composite unit. Three bags are three iterations of that unit, or 12 apples. A ratio table records the coordination of these two quantities.

In interviews with 24 sixth graders before instruction, unitizing and relative thinking were associated with more sophisticated strategies (Lamon, 1993). Thompson and Saldanha (2003) describe multiplicative reasoning as reasoning about how many times as large one quantity is as another. In a ratio table, each column pairs records the number of units with the quantity those units compose.

Students need to understand the unit relationship and be able to reason back to it. Writing the column for 1 makes that relationship explicit early in learning. Later, students reason from any known pair. A student who knows that 4 corresponds to 20 can reason to 2 and 10, 1 and 5, 8 and 40, or 12 and 60.

> **Key Idea • Students need to understand the unit relationship and be able to reason back to it.**

Iteration, Composition, and Scaling

Students' reasoning with a unit becomes more efficient and more flexible over time. Drawing from this research and DMTI's work with learning progressions, we describe the development as follows:

Iterate a unit → compose units → decompose quantities → relationships • scale quantities → coordinate two quantities proportionally.

A student who iterates adds one unit at a time: 3, 6, 9, 12. A student who composes combines known quantities: five units and two units compose seven units. A student who decomposes partitions a target quantity into known quantities: 13 units are 10 units and 3 units. A student who scales multiplies or divides both quantities by the same factor: if 4 corresponds to 20, then 8 corresponds to 40. Fosnot and Dolk (2001) describe similar movement in students' multiplication strategies, from counting and repeated addition toward strategies that use relationships among known products.

This is a developmental progression, not a rigid sequence. Noelting (1980) described students first comparing within a single quantity and only later coordinating two quantities at once. However, Karplus et al. (1983) found that 11- and 13-year-olds' strategies varied more with the numbers and the context of a problem than with age. Students move back and forth among these ways of reasoning as the numbers and contexts change.

> **Key Idea • Students iterate a unit, compose units, decompose quantities and relationships; scale quantities. This is a developmental progression, not a rigid sequence.**

Ratio Tables as Representations of Multiplicative Relationships

A bar model can make quantities and their relative sizes visible. A double number line coordinates two quantities along continuous scales. A ratio table organizes corresponding quantities numerically. Each representation makes different aspects of the relationship available for reasoning.

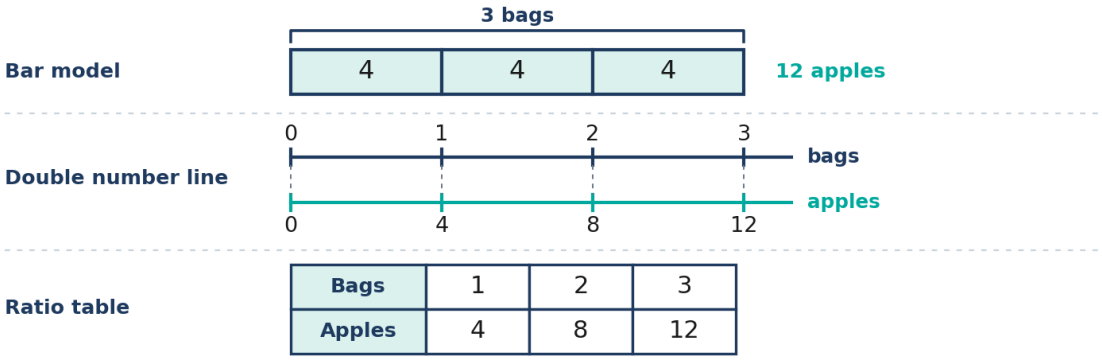


Figure 1. One relationship, 4 apples in each bag, in three representations.

Lesh et al. (1987) describe understanding as including the ability to translate among representations of the same idea. Symbolic representations allow students to see the same multiplicative relationship across them. A bar model that shows three units of four and a table column that pairs 3 with 12 record one relationship in two forms. Gravemeijer (1999) describes how a model of a particular situation can become a model for more general mathematical reasoning.

When you graph corresponding values from a proportional relationship with the independent quantity on the horizontal axis and the dependent quantity on the vertical axis, the points lie on a line through the origin. The slope of that line represents the constant of proportionality. The ratio table contains two quantities, so the constant of proportionality depends on which quantity is treated as dependent: 4 apples per bag in one direction, and one-fourth of a bag per apple in the other.

The equation $y = kx$ expresses the relationship that the table records, with k as the constant of proportionality. Ratio tables also extend beyond whole numbers. Brendefur and Pitingoro (1998) describe middle school students using a ratio table to divide fractions.

> **Key Idea • A representation earns its place when it makes the multiplicative relationship visible.**

What Student Responses Can Reveal

These responses can reveal the relationship a student is using. An additive response may indicate that the student is preserving a difference rather than a multiplicative factor. Sequential counting may show that the student can iterate a unit but has not yet begun composing larger units efficiently.

- **A student records 1 and 4, 2 and 6, 3 and 8.** The student may be preserving an additive difference rather than a multiplicative relationship.
- **A student records a column that does not fit the others.** The student may have lost the unit relationship and may not be checking a new column against a known pair.
- **A student counts every column from 1 to reach 37.** The student can iterate a unit but may not yet compose or decompose quantities.
- **A student reasons with whole numbers but not with fractions, decimals, or rates.** The student's reasoning may still be tied to particular numbers or contexts. Students reason multiplicatively more often when ratios are whole numbers (Van Dooren et al., 2010).

A response is evidence, not a diagnosis. The same written table can result from different reasoning, and a correct table does not necessarily show that a student is reasoning multiplicatively. Asking students to explain what stays the same to the next gives teachers more information than the completed table alone. Assessments that distinguish additive errors from multiplicative errors provide information that a count of correct answers does not (Hilton et al., 2013).

> **Key Idea • A response can reveal the relationship a student is using.**

Knowledge Organization and Transfer

What students already understand affects what they can attend to in a new problem. When multiplication facts and procedures are connected to relationships among quantities, students have structures they can draw on in unfamiliar situations. When knowledge remains tied to particular procedures or problem forms, transfer is more difficult.

Research on learning shows that knowledge organized around central ideas is more readily retrieved and applied in new settings than knowledge held as separate facts (Bransford et al., 2000). Hiebert and Carpenter (1992) describe mathematical understanding in terms of the number and strength of connections among ideas, facts, and procedures. Skemp (1976) distinguished relational understanding, knowing both what to do and why, from instrumental understanding, knowing rules without reasons.

Cognitive load theory addresses a narrower question. Working memory is limited, and attention spent on unnecessary demands is not available for learning (Sweller, 1988). A table that keeps corresponding quantities aligned may reduce what students must hold in mind while they coordinate two quantities. This is an application of the theory to ratio tables, not a finding about them.

> **Key Idea • Knowledge connected to relationships among quantities is more available in unfamiliar situations than knowledge tied to particular procedures.**

What the Research Suggests for Instruction

Research supports instruction that develops multiplicative relationships, connects representations, attends explicitly to units, and gives students opportunities to reason across equivalent ratios rather than rely only on formal procedures. Lamon (2007) observes that instruction has traditionally emphasized procedures, cross multiplication in particular, over the reasoning behind them. A federal practice guide recommends developing ratio and rate reasoning before teaching cross multiplication, and rates the evidence for that recommendation as minimal (Siegler et al., 2010).

The evidence has limits. In a randomized study of 806 seventh graders with problem-solving difficulties, instruction organized around the structure of ratio and proportion problems improved proportional problem solving, and the gains held nine weeks later (Jitendra et al., 2017). However, we did not locate a controlled study that isolates the effect of ratio tables on students' proportional reasoning. The rationale for using ratio tables draws on research on multiplicative reasoning, unitizing, and representation, as well as instructional design.

DMT Companion 47 describes an instructional sequence, questions for teachers and coaches, and what to look for in student work.

> **Key Idea • Develop multiplicative relationships, connect representations, and attend explicitly to units.**

Conclusion

A ratio table records multiplicative reasoning. It does not produce that reasoning. A student's use of the table can provide evidence of whether the student preserves a difference or a factor, iterates or composes units, and reasons from known relationships.

The research reviewed here addresses the same ideas from several directions: the shift from additive to multiplicative reasoning, the coordination of composite units, the connections among representations, and knowledge organized around relationships. The relationships students record in a table in the elementary grades are the relationships they later express as unit rates, proportional relationships, and y = kx.

> **Key Idea • Students need to understand the unit relationship and be able to reason from any known pair.**

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Social Media

A student builds a ratio table for 4 apples in each bag and records 1 and 4, 2 and 6, 3 and 8. What relationship is the student using?

The student has kept a difference of 2 from one column to the next. That reasoning works in many earlier situations. Here the two quantities are related by a factor of 4. Fact knowledge and multiplicative reasoning are related, but they are not the same. Inside this DMT Insight, we look at:

- How students move from additive to multiplicative reasoning
- Why unitizing and composite units matter
- What a bar model, a double number line, and a ratio table each make visible
- What the research does and does not show about ratio tables

Read the full Insight at mathsuccess.io/dmtinsights.

What do your students keep the same from one column to the next?

