

DMT INSIGHTS

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RATIO TABLES: BUILDING MULTIPLICATIVE AND PROPORTIONAL REASONING

Introduction

Why do students who can recite their multiplication facts still fall apart when they hit division, fractions, decimals, and proportions—and what does a ratio table have to do with it?

Students who can correctly recall multiplication facts often fall apart when mathematics becomes more complex. **This is not a student effort issue—it is an instructional design issue. Students do not understand the relationships behind them.** When multiplication is taught as fact retrieval rather than relational thinking, students lack a framework for scaling, rates, and proportions. The tools that could build that foundation—visual models, structured tables, flexible strategies—are frequently skipped in favor of procedures that produce answers without understanding (Lamon, 2007).

Ratio tables are not a worksheet activity or a shortcut for computation. **They are a symbolic structure that makes multiplicative relationships visible, flexible, and transferable.** Ratio tables are a flexible algorithm students can use to solve multiplicative relationships from grade 3 to algebra—they are a structure students learn to think with. This DMT Insight examines what ratio tables are, why students need them, and what research tells us about teaching them effectively.

The Core Problem—Procedural Multiplication Without Relational Structure

What happens in classrooms where multiplication means memorizing facts rather than understanding relationships?

The central instructional problem is a failure to develop **multiplicative reasoning**—the ability to reason about relationships among quantities, rather than just to retrieve isolated facts.

- Cannot scale efficiently (e.g., move from 4×3 to 40×3 or 17×3)
- Rely on counting strategies (1, 2, 3, 4...) well beyond primary grades

- Struggle with missing value problems
- Fail to transfer knowledge to division, decimals, fractions, ratios, and algebra

From a cognitive psychology perspective, fact-first instruction imposes high extraneous cognitive load: students juggle memorized answers without a coherent mental model of what those answers represent, making transfer to new contexts extremely difficult (Sweller, 1988; Hiebert & Carpenter, 1992). **The result: students can compute, but they cannot reason.** These patterns are not accidental. They point directly to why ratio tables—and the way they are taught—matter.

Ratio Tables Are Symbolic Models—Not Visual Models

How do ratio tables fit into a broader learning progression—and where do they ultimately lead?

A bar model shows the size of a quantity. A ratio table shows the relationship between quantities. Ratio tables are symbolic structures that organize relationships numerically.

That is why **visual models must come first or alongside them**. Bar models and double number lines help students see and feel equal units (groups) and multiplicative relationships. Ratio tables then encode those relationships symbolically, requiring students to coordinate values abstractly (Lesh, Post, & Behr, 1987).

This progression is not a one-time sequence. Visual models should continue alongside ratio tables when introducing new contexts or addressing misconceptions. The goal is integration, not replacement. The endpoint of this progression is algebraic graphing. The coordinate pairs in a ratio table, when plotted, produce a straight line through the origin. **The constant in a ratio table is the slope of the line.**

This makes ratio tables a direct precursor to $y = mx$, where **m** is the unit rate students have been reasoning about. Proportional reasoning, rate, and linear functions are not separate topics—they are one coherent idea expressed in increasingly formal representations. Understanding ratio tables as symbolic structures—rather than visual aids or computation shortcuts—is what allows teachers to use them as a genuine bridge to algebra.

Predictable Misconceptions and Their Instructional Roots

What are the most common errors teachers see—and what does typical instruction do to produce them?

Research and classroom evidence reveal a consistent pattern of errors when multiplicative reasoning is underdeveloped (Carpenter, Fennema, & Franke, 1996; Fosnot & Dolk, 2001):

- **Additive Thinking:** Students add a constant instead of maintaining a multiplicative relationship ($1 \rightarrow 4$, $2 \rightarrow 6$, $3 \rightarrow 8$)
- **Losing the Unit:** Students lose the “1 column” anchor and produce values that break equivalence
- **Sequential Counting:** Students rely on counting (1, 2, 3, 4...) even for large numbers like 37×6

- **Lack of Transfer:** Students cannot extend reasoning to fractions, decimals, or rates

These are not random mistakes—**they are predictable outcomes of instruction.** These errors arise when multiplication is treated as fact memorization before structural understanding is established, and when ratio tables—if they appear at all—are treated as fill-in-the-blank activities rather than flexible reasoning tools.

Misconceptions as Diagnostic Feedback

*What are student errors **actually telling** us—and how should that change what we do next?*

Student errors are not just mistakes—**they are evidence of how instruction shaped students’ thinking.** Additive errors ($1 \rightarrow 4$, $2 \rightarrow 6$, $3 \rightarrow 8$) signal that students were never asked to distinguish between adding a constant and maintaining a multiplicative relationship. Over-reliance on sequential counting signals that instruction did not help students see that larger numbers can be composed from known parts.

Failure to transfer signals that ratio tables were taught as a procedure for a specific context, not as a generalizable structure. These patterns collectively indicate that instruction introduced symbolic shortcuts before conceptual understanding was established, and failed to make the unit relationship explicit and central (Gravemeijer, 1999; Lesh et al., 1987). When teachers read errors as diagnostic signals rather than simple wrong answers, instruction becomes more precise, more responsive, and more effective.

The Developmental Trajectory: From Iteration to Flexible Reasoning

How does ratio table thinking develop over time—and what should teachers do at each stage to move students forward?

When students first encounter ratio tables, they naturally begin sequentially—starting at 1 and building to 2, 3, 4. This is developmentally appropriate and should be connected to the construction of bar models. The 1–4 building phase gives students a firm anchor in the unit relationship. This early phase is where students establish **what “1” represents—the foundation of all multiplicative reasoning.** Rushing past it, or treating the sequential table as a mere warm-up, undermines everything that follows.

What distinguishes skilled multiplicative reasoners is the move toward **flexibility:** scaling by landmark numbers ($\times 5$, $\times 5$, $\times 10$), decomposing non-landmark numbers ($13 = 10 + 3$), and composing known columns to find unknown ones. Fosnot and Dolk (2001) describe this as the shift from additive to multiplicative thinking—one of the most significant conceptual leaps in elementary mathematics. Lamon (2007) emphasizes that this shift requires explicit instruction in unitizing: students must learn to visualize numbers not just as counts but as composed units that can be strategically manipulated.

This progression does not happen automatically. **Without explicit teacher prompts—“How can you jump to 10? How can you decompose 7?”—students remain stuck in sequential counting well into middle school.** The flexibility

has to be taught.

A Coherent Instructional Pathway

*What does a well-designed instructional sequence **actually look like**—and how does it hold together across grades?*

The goal is not a single good lesson—it is **instructional coherence across grades**, where each phase builds directly on the last. Research points to **five interlocking moves** that, taken together, build durable multiplicative understanding:

First, build meaning through iconic models before introducing the ratio table. Bar models and double number lines give students a visual experience of the unit relationship—what one unit (group) is worth—before any symbolic encoding begins.

Second, anchor every table to the unit column. Introduce the table starting from 1, and make that anchor explicit and non-negotiable: every other column must be verifiable against it.

Third, develop flexibility explicitly rather than waiting for it to emerge. Prompt students to iterate through 2, 3, 4, and 5, then look for patterns. Then decompose non-landmark numbers. This shift—from counting up to composing strategically—is an example of multiplicative reasoning.

Fourth, generalize the structure by introducing the 1–2–5–10 landmark table as the full flexible algorithm. Once students can build and use landmark columns fluently, they have a tool that works for whole numbers, decimals, fractions, and rates.

Fifth, delay formal algorithms until understanding is established. Cross multiplication is efficient—but only for students who already understand what a proportional relationship is. Introducing it too early replaces reasoning with rule-following (Skemp, 1976).

Detailed classroom tasks, lesson sequences, and teacher moves for each of these phases are available in the DMTI companion guide. This pathway works because it **manages cognitive load deliberately**: concepts are sequenced logically, visual tools reduce abstraction at each new stage, and symbolic reasoning is built on a foundation of relational understanding rather than imposed on top of memorized facts (Sweller, 1988).

Conclusion

The persistent difficulties students experience with proportional reasoning are not a mystery, nor are they the result of students not trying hard enough. **These difficulties are the predictable outcome of instruction that introduces symbolic shortcuts before conceptual understanding is established.** When multiplication is taught through ratio tables—grounded in the unit relationship, supported by visual models, and developed toward flexible strategies—students build a durable understanding that transfers across contexts.

The same structure that helps a third grader reason about equal groups helps a sixth grader reason about unit rates

and a seventh grader understand proportional relationships. **One structure, taught well, carries students from Grade 3 to algebra.** Ratio tables are a bridge from arithmetic to algebra, from equal groups to linear functions. Teaching them well requires an instructional design that makes structure visible, honors developmental progression, and insists that every student understand not just *what* the answer is, but *why* the relationship holds. **This is how we move from students getting answers to students understanding mathematics.**

References

- Brendefur, J. L., & Pitingoro, L. (1998). Dividing fractions using the ratio table. *Mathematics Teaching in the Middle School*, 4(2), 122–127.
- Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction. *The Elementary School Journal*, 97(1), 3–20.
- Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Heinemann.
- Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning*, 1(2), 155–177.
- Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 65–97). Macmillan.
- Lamon, S. J. (2007). Rational numbers and proportional reasoning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 629–667). Information Age.
- Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations. In C. Janvier (Ed.), *Problems of representation in the teaching and learning of mathematics* (pp. 33–40). Erlbaum.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.
- Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. *Cognitive Science*, 12(2), 257–285.

Social Media Post

Why do students who know their times tables still fall apart when they hit fractions, rates, and proportions?

Because memorizing facts is not the same as understanding relationships. Our latest DMT Insight shows:

- Ratio tables are a relational structure—not a worksheet activity
- The unit relationship (“what does 1 represent?”) is the foundation everything else rests on
- Sequential counting (1, 2, 3, 4) is a starting point, not a destination
- The same structure that builds multiplication in Grade 3 leads directly to $y = mx$ in Grade 7

This isn’t a student problem—it’s an instruction problem.

When teachers ground ratio tables in the unit relationship, explicitly develop flexibility, and connect tables to graphs and linear functions, students understand, and that understanding transfers. Check out our work at www.dmtinstitute.com and MathSuccess.io.

What's one place your students get stuck moving from multiplication to proportional reasoning? Share at contact@dmteinstitute.com

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