

TEACHING QUADRILATERALS FOR UNDERSTANDING: BEYOND “NAME THE SHAPE”

Classifying quadrilaterals from properties and relationships

Introduction

Why do so many students insist that a square cannot be a rectangle, and what does that tell us about how we teach quadrilaterals?

These responses are not random. They often reflect how quadrilaterals have been presented. In many classrooms, learning about quadrilaterals centers on naming shapes and matching words to pictures. Students learn to recognize what shapes look like, not what must be true about them. They rarely investigate what must always be true about a shape, regardless of how it is oriented or drawn.

Quadrilaterals give students an important opportunity to reason about attributes, classification, and logical relationships. When instruction develops these relationships, students move beyond naming shapes to reasoning about the properties that define them and how categories relate.

Research in mathematics education suggests that students need to move beyond recognizing shapes by appearance toward reasoning about the properties and relationships that define them (Clements & Battista, 1992; de Villiers, 1994; Fujita & Jones, 2007). This DMT Insight examines why students classify shapes by appearance and how instruction can shift their attention to defining properties and relationships.

Key Idea • Quadrilaterals should be understood through their properties and relationships, not simply by what they look like.

The Problem: Picture-Based Geometry

When geometry instruction centers on memorizing the appearance of shapes, students can develop what researchers call prototype-based reasoning. They form images of a category from repeated examples, such as the upright triangle, the horizontal rectangle, and the square resting flat on a side, and judge whether a shape belongs to a category by how closely it resembles those examples (Rosch, 1978). The result is a geometry of appearances rather than a geometry of properties.

Researchers have documented the results. Students often:

- Reject a rotated square as “not a real square” or label it a diamond
- Insist that a square is not a rectangle because rectangles are “the long ones”
- Fail to recognize non-prototypical examples, such as an irregular quadrilateral or a non-isosceles trapezoid
- Accept shapes as belonging to a category based on overall appearance rather than defining properties

These responses can reveal how students are thinking about the category. When instruction emphasizes what shapes look like, students may use appearance rather than defining properties to classify them. In their review of the research, Clements and Battista (1992) report that many students do not accept a square as a rectangle. De Villiers (1994) distinguishes a hierarchical classification, in which one category is included in another, from a partition classification, in which the categories do not overlap, and describes students’ difficulty with the hierarchical classification.

The same pattern appears in everyday classroom materials. Worksheets show parallelograms leaning only to the right. Definitions describe rectangles as “longer than they are tall.” Repeated exposure to these examples can reinforce the misconception. Young children identify typical examples of a shape more readily than atypical ones (Dagli & Halat, 2016; Verdone et al., 2016).

Key Idea • When students repeatedly see only prototypical examples, appearance can become part of what they think defines the shape.

Why Students Rely on Familiar Examples

Research also helps explain why students may keep relying on appearance even after learning definitions. Two ideas are especially useful: prototype effects and the distinction between concept image and concept definition.

Prototype effects describe the tendency to decide whether a shape belongs to a category by comparing it with a familiar example rather than checking its defining properties (Rosch, 1978). In geometry classrooms, the typical rectangle is horizontal and longer than it is tall, and the typical square has a horizontal base. When students encounter atypical examples, such as a tall, narrow rectangle or a rotated square, they often reject them because the shapes do not match the familiar example, even though the shapes have the defining properties.

The distinction between concept image and concept definition helps explain why this problem persists even after formal instruction (Vinner, 1991). A concept image includes the pictures, examples, and ideas a student associates with a shape. A concept definition states the properties that define the category.

Students may state a definition correctly and still classify a shape by its appearance. This helps explain why definitions alone are not enough. Students need repeated opportunities to apply defining properties to varied examples and non-examples.

The examples need to vary in orientation, size, and dimensions so that students can distinguish defining properties from features that can change.

Key Idea • Students may be able to state a definition correctly and still classify a shape by its appearance.

The Mathematical Structure: Lines, Properties, and Hierarchies

A quadrilateral is a polygon with four sides. Each side is a line segment. Students can extend those segments conceptually as lines to reason about relationships such as parallelism and perpendicularity. The properties that distinguish quadrilateral categories are relationships among the sides and angles: which sides are parallel, which angles are right angles, and which sides are congruent.

Each category is defined by its properties:

- **Parallelogram:** a quadrilateral with two pairs of parallel sides.
- **Rectangle:** a parallelogram with four right angles.
- **Rhombus:** a parallelogram with four congruent sides.
- **Square:** a parallelogram with four right angles and four congruent sides.

A square satisfies all of the defining properties of a rectangle and has additional properties. That is why every square is also a rectangle. For the same reason, every square is also a rhombus. The categories are inclusive, not mutually exclusive.

In this document, a trapezoid is defined inclusively as a quadrilateral with at least one pair of parallel sides. Under this definition, every parallelogram is also a trapezoid. A kite is a quadrilateral with two pairs of consecutive congruent sides, where the sides of one pair are not congruent to the sides of the other pair. Under this definition, a rhombus is not a kite.

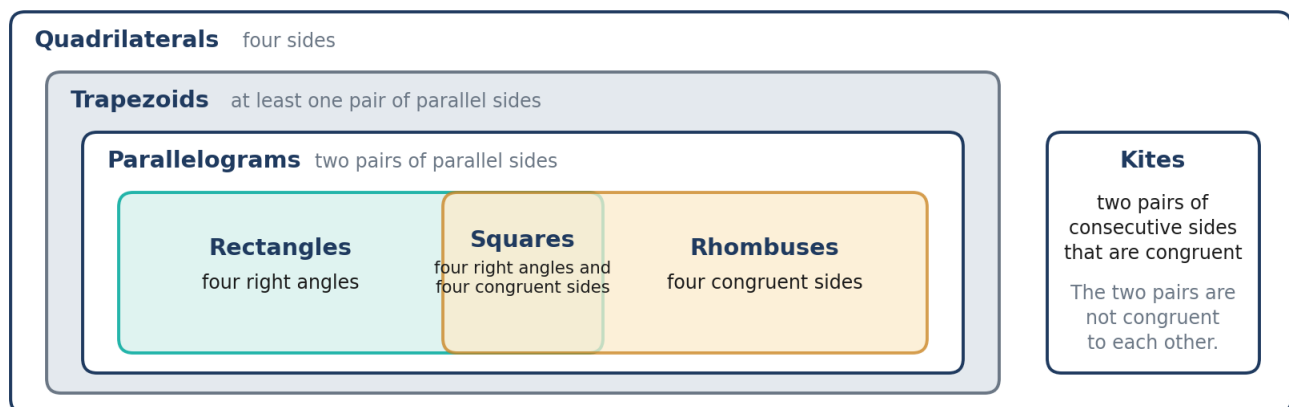


Figure 1. How quadrilateral categories are related.

These relationships give students something to reason from when a shape changes orientation or appearance. Orientation does not change them: two parallel sides remain parallel when the figure is rotated. The categories connect to their defining properties rather than their typical appearance. A student can look at a rotated rectangle and ask, “Are the opposite sides parallel? Are the angles right angles?” rather than asking, “Does this look like a rectangle?”

Fujita and Jones (2007) describe how learners’ personal images of quadrilaterals, often limited to prototypical examples, can obstruct understanding of how one category fits within another. Clements and Sarama (2009) describe learning trajectories in which children move from recognizing shapes as wholes to identifying and reasoning about their properties.

Key Idea • A square satisfies all of the defining properties of a rectangle and has additional properties. That is why every square is also a rectangle.

How Students' Geometric Reasoning Develops

Students often begin by recognizing shapes from their overall appearance. With instruction, they can attend to specific properties and eventually reason about relationships among categories (Clements & Battista, 1992). For example, students move from recognizing a square by sight to explaining why every square also satisfies the definition of a rectangle.

The sides of a quadrilateral are line segments. Extending those segments as lines can help students reason about parallel and perpendicular relationships. A line extends in both directions without end. Parallel lines never meet, no matter how far they are extended. Perpendicular lines meet at a right angle. These relationships give students a basis for explaining how quadrilateral categories are defined and how they are related.

This progression does not happen automatically. Without instruction that moves students from appearance to properties and relationships, many students continue to classify by appearance into the middle grades (Clements & Battista, 1992). Presenting definitions is not enough. Students need varied examples and non-examples, sorting tasks that require justification, and opportunities to reason about which properties define a category and why some shapes belong to more than one.

Understanding that every square is a rectangle requires students to see that one category can be included in another. This is difficult for students who have learned to treat shape names as mutually exclusive labels. The understanding develops gradually. It requires instruction that addresses the relationships among categories, not only the vocabulary.

Key Idea • As students' geometric reasoning develops, they increasingly use properties and relationships to classify shapes and explain how the categories are related.

What This Means for Instruction

Several instructional practices can help students move from classifying shapes by appearance to reasoning from defining properties.

- **Investigate properties and relationships.** Give students opportunities to examine line segments, angles, parallel relationships, perpendicular relationships, and congruent sides as they classify shapes.
- **Ask, "What must be true?"** Ask which properties a shape must have to belong to a category, not what the shape looks like.
- **Use examples and non-examples.** Include non-examples that have some, but not all, of the defining properties.
- **Vary orientation and dimensions.** Include rotated, stretched, and non-prototypical examples so that students can distinguish defining properties from features that can change.
- **Compare and classify shapes.** Ask students to sort shapes and to explain what the shapes in a category have in common.
- **Make the hierarchy explicit.** Use nested diagrams to show how one category is included in another.

- **Ask students to justify their classifications.** Questions such as “How do you know?” and “What would have to change for this to become a square?” show whether a student is reasoning from properties or from appearance.

DMT Companion 46 describes teacher moves, questions, and common misconceptions. The DMTI Quadrilateral Poster, available on Teachers Pay Teachers and Redbubble, shows the hierarchy for classroom use.

Key Idea • Ask what must be true, vary the examples, and ask students to justify their classifications.

Conclusion

Quadrilateral instruction should move beyond naming shapes. Students need opportunities to examine properties, compare examples and non-examples, and reason about how shape categories are related.

A square is not a rectangle because a diagram tells students where to place it. A square is a rectangle because it satisfies the defining properties of a rectangle. When students can explain that relationship, they are reasoning from the properties that define the categories rather than from appearance.

The question “What must be true?” helps shift the work from naming a shape to reasoning about why it belongs to a category.

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Social Media

Why do so many students insist that a square is not a rectangle?

Many students classify shapes by what they look like, not by the properties that define them. The examples students see, and the questions teachers ask, influence how students learn to classify shapes. Inside this DMT Insight, we look at:

- Why students classify quadrilaterals by appearance
- How repeated prototypical examples can influence the way students classify shapes
- How reasoning from properties helps students classify shapes across different orientations
- Why every square is also a rectangle, and why definitions alone are not enough

Read the full Insight at mathsuccess.io/dmtinsights. A classroom quadrilateral poster is available on Teachers Pay Teachers and Redbubble.

What quadrilateral misconception do you see each year?

