

# DMT INSIGHTS

Issue 45 · 2026 · by Jonathan Brendefur, PhD, and Associates



## WHY SUBTRACTION IS NOT JUST ADDITION BACKWARD

*Reasoning about quantities, differences, and missing quantities*

### Introduction

Students can compute subtraction correctly yet still struggle to explain what the difference represents. A student may solve  $26 - 17$  accurately in one situation but struggle when the same quantities appear in a comparison or missing-quantity problem.

Research on addition and subtraction shows that these operations involve more than just calculation. Students must coordinate the quantities in a situation, determine how those quantities are related, and identify what is unknown (Fuson, 1992; Nunes & Bryant, 1996; Verschaffel et al., 2007).

Subtraction involves more than taking away or following a procedure. Students must recognize how quantities are related, determine what is unknown, and connect those relationships to models, equations, and increasingly efficient procedures.

### The Mathematical Structure of Subtraction

Subtraction involves a relationship among quantities, but that relationship can take different forms. A quantity may be removed from a total, an unknown quantity may complete a total, or two quantities may be compared to determine their difference.

The equation  $26 - 17 = 9$  can therefore represent different situations. The symbols alone do not tell us what 26, 17, and 9 represent. The situation determines what each quantity represents.

Research on addition and subtraction problem types distinguishes several problem structures (Carpenter et al., 1996; Fuson, 1992; Verschaffel et al., 2007). Three especially important subtraction situations are:

- **Take-away:** a quantity is removed, and the remaining quantity is unknown.
- **Missing quantity:** a total and one quantity are known, and another quantity must be determined. The research literature calls this a missing-addend problem.
- **Comparison:** two quantities are known, and the difference between them is unknown.

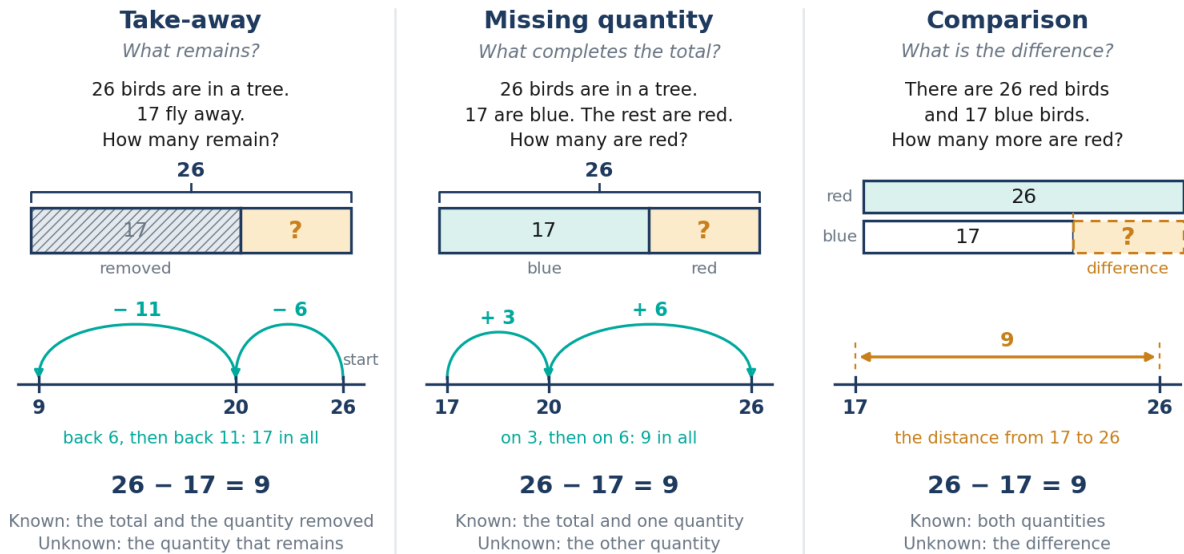


Figure 1. The same numbers can represent different subtraction situations. The models make the relationship among the quantities visible.

Carpenter and Moser (1984) found that young children used different strategies depending on the subtraction situation. Students separated quantities in take-away problems, added on in missing-quantity problems, and matched quantities in comparison problems. Selter et al. (2012) further examined taking away and determining the difference as two important models of subtraction.

A student who knows only a procedure may calculate the difference without recognizing how the quantities in the situation are related. Connecting equations, situations, and models gives the symbols something to represent (Hiebert & Carpenter, 1992).

**Key Idea • The same equation can represent different relationships. The situation determines what each quantity represents.**

## What Student Errors Can Reveal

Student errors in subtraction are often systematic. One well-documented error is the “smaller-from-larger” error, in which a student subtracts the smaller digit from the larger digit in each place, regardless of which quantity is being subtracted (for example,  $274 - 138 = 144$ ). Brown and VanLehn (1982) describe errors of this kind as “bugs”: consistent procedures that a student constructs on reaching a step the student does not know how to complete. This error can occur when a student treats subtraction as a digit-by-digit procedure rather than reasoning about the value of the units in each place.

An error in a word problem can reveal something different. A student who reads “How many more does Maria have?” and adds 26 and 17 is choosing an operation from a word in the problem, not from the relationship between the two quantities.

Each of these responses provides evidence of the relationship or procedure a student is using:

- A student writes  $274 - 138 = 144$ . The student may be subtracting digit by digit without attending to the value of the units in each place.
- A student writes  $17 - 26 = 9$ . The student may be treating subtraction as if the order of the quantities does not matter.
- A student solves a take-away problem but not a comparison problem with the same numbers. The student may be interpreting subtraction only as removal.

A response is evidence, not a diagnosis. The same written answer can result from different reasoning. Asking the student to explain what each quantity represents gives more information than the answer alone.

**Key Idea • An incorrect answer can provide evidence of how a student is reasoning. Ask what relationship or procedure produced the answer before correcting it.**

## Cognitive Demands of Subtraction

Multi-step subtraction can strain working memory as students coordinate place-value units, decompositions, and intermediate results (Geary, 2011; Sweller, 1988). As number relationships and procedures become more efficient, students can devote more attention to the problem's mathematics.

Magnitude also matters. Subtraction can represent the distance between two quantities, not only the removal of one quantity from another. A number line shows this relationship because the difference is the distance between two positions. Children's knowledge of numerical magnitude develops through experience with linear representations: low-income preschoolers who played a linear number board game improved in number line estimation and magnitude comparison, while those who played a circular version did not (Siegler & Ramani, 2009).

## Connecting Meaning, Models, and Procedures

- **Connect situations, models, and equations.** Students should be able to explain what each quantity represents and how the model shows the relationship.
- **Vary the problem structure.** Include take-away, missing-quantity, and comparison situations so subtraction is not reduced to removal.
- **Connect addition and subtraction.** Equations such as  $17 + 9 = 26$  and  $26 - 17 = 9$  describe the same relationship among quantities from different perspectives.
- **Connect procedures to place-value relationships.** In multi-digit subtraction, students decompose units into equivalent quantities. One ten can be decomposed into ten ones without changing the total quantity. The written algorithm records these relationships efficiently.

Procedures should be connected to relationships students can reason about. Research describes conceptual and procedural knowledge as developing together, with gains in one supporting gains in the other (Rittle-Johnson et al., 2015).

Bar models and number lines make different relationships visible. A bar model can show a total and the quantities that compose it, or two quantities and their difference. A number line can show the difference as the distance between two positions. A model earns its place when it makes the relationship among the quantities visible. Students can move among situations, models, and equations in any order as understanding and fluency develop together.

For elementary students at risk for mathematics failure, a single-case study found that instruction connecting concrete materials, drawings, and written notation was functionally related to improved subtraction with regrouping (Flores, 2010).

## Multi-Digit Subtraction and Place Value

Multi-digit subtraction depends on place-value relationships (Fuson, 1992). When a unit in one place is decomposed, the quantity remains the same: one hundred can be decomposed into ten tens, and one ten into ten ones. The standard subtraction algorithm records these decompositions efficiently.

Students should be able to connect the written notation to what happens to the units. Rather than saying that a digit is “borrowed,” ask: What unit was decomposed? How many ones are there now? Did the total quantity change?

**274**



↓ **decompose 1 ten into 10 ones**



**$200 + 60 + 14 = 274$ . The quantity is equal.**

**The written procedure records the decomposition**

|       |          |              |              |                                    |
|-------|----------|--------------|--------------|------------------------------------|
|       |          | <b>6</b>     | <b>14</b>    |                                    |
|       | 2        | <del>7</del> | <del>4</del> | 14 ones – 8 ones = 6 ones          |
| –     | 1        | 3            | 8            | 6 tens – 3 tens = 3 tens           |
| <hr/> |          |              |              |                                    |
|       | <b>1</b> | <b>3</b>     | <b>6</b>     | 2 hundreds – 1 hundred = 1 hundred |

*What unit was decomposed? Did the total quantity change?*

Figure 2. In  $274 - 138$ , one ten is decomposed into ten ones. The quantity remains the same, and the written procedure records the decomposition.

In a three-year longitudinal study of 82 children, students who used invented strategies before learning standard algorithms showed stronger knowledge of base-ten number concepts and were more successful at extending that knowledge to new situations (Carpenter et al., 1998).

**Key Idea • The written procedure records what happens to the units. Ask what unit was decomposed and whether the total quantity changed.**

## Conclusion

Subtraction describes relationships among quantities. Students may be finding what remains, determining a missing quantity, comparing two quantities, or using place-value relationships to calculate a difference.

These relationships should be visible in the situations students solve, the models they use, and the procedures they develop. Students need to understand what the quantities represent and how they relate.

The goal is not to choose between understanding and procedural fluency. Students need efficient procedures and mathematical relationships they can reason from. When students forget a step, understanding gives them something to rebuild from.

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## Social Media

A student solves  $26 - 17$  correctly. Can the student explain what the 9 represents?

The equation  $26 - 17 = 9$  can represent what remains, a missing quantity, or the difference between two quantities. The equation is the same. What the quantities represent is different. Inside this DMT Insight, we look at:

- Three subtraction situations: take-away, missing quantity, and comparison
- What student errors can reveal about the relationship or procedure a student is using
- What working memory and magnitude have to do with subtraction
- How place value connects the written procedure to the quantities

Read the full Insight at [mathsuccess.io/dmtinsights](https://mathsuccess.io/dmtinsights).

What do your students say the difference represents?

