

DMT INSIGHTS

Issue 44 · 2026 · by Jonathan Brendefur, PhD, and Associates



UNDERSTANDING DEPTH OF KNOWLEDGE IN MATHEMATICS

What the research suggests about cognitive demand, mathematical reasoning, and instruction

Introduction

Depth of Knowledge (DOK), developed by Webb (1997, 2002), provides a framework for examining the cognitive complexity of tasks and assessments. DOK is sometimes interpreted as a measure of difficulty, but these are different considerations. A task may be difficult because it involves unfamiliar numbers or a lengthy procedure without requiring more complex reasoning.

In mathematics, the DOK level depends on what students must know and do to complete a task. Retrieving a fact, applying a procedure, interpreting quantities in a context, representing a mathematical relationship, and developing a justification can involve different cognitive demands. You cannot determine the level solely by the size of the numbers, the number of steps, or whether a task includes a diagram or explanation.

This distinction matters when examining curriculum, instruction, and assessment. Instruction should give students opportunities to develop procedural fluency, interpret quantities, represent mathematical relationships, and justify their reasoning. DOK can help educators evaluate the kinds of mathematical thinking their tasks require.

Cognitive psychology research also offers additional considerations for instruction. Retrieval practice supports retention (Roediger & Karpicke, 2006), and prior knowledge shapes a task's working-memory demands (Sweller, 1988). When selected and used appropriately, representations and explanations can help students examine mathematical relationships. These findings inform instructional design, but they do not determine a task's DOK classification.

Key Idea • DOK describes the cognitive complexity of the mathematical work a task requires. It does not measure the difficulty of the numbers or how long the task takes.

DOK 1: Recall and Routine Procedures

DOK 1 generally involves recalling information, recognizing mathematical facts, or carrying out routine procedures. In multiplication, this might include recalling that $7 \times 8 = 56$ or applying a practiced algorithm. DOK 1 work is essential. Retrieval practice supports long-term retention (Roediger & Karpicke, 2006), and well-practiced facts and procedures place fewer demands on working memory (Willingham, 2009). As facts and procedures become more fluent, students can use them more efficiently when solving problems that require additional reasoning. Increasing the size of the numbers does not, by itself, change the DOK level: a page of larger multiplication problems remains DOK 1 when

students are carrying out a familiar procedure. Conceptual understanding and procedural fluency develop together (Rittle-Johnson & Siegler, 1998). Instruction should help students understand mathematical relationships while developing accuracy and efficiency with facts and procedures.

DOK 2: Applying and Representing Mathematical Relationships

DOK 2 involves applying mathematical knowledge, interpreting quantities, selecting strategies, and representing relationships. For instructional purposes, DMTI distinguishes two common forms of mathematical work within DOK 2: interpreting quantitative relationships in context and representing mathematical relationships through models. These are instructional distinctions, not separate levels in Webb’s original framework.

DOK 2: Contextual Interpretation

A contextual task may require students to identify relevant quantities, determine how those quantities are related, and select an appropriate operation or strategy. When the relationship is not immediately specified, and students must interpret the situation, the task may involve DOK 2 reasoning. A familiar problem such as “There are 7 rows of chairs with 8 chairs in each row. How many chairs are there?” may require only routine application for students who have practiced that structure many times. Research on word problems describes how students must connect the language of a situation to a quantitative relationship (Kintsch & Greeno, 1985). Contextual tasks should assess the mathematical reasoning they are intended to assess, not reading complexity.

DOK 2: Mathematical Representation and Modeling

Students may represent a multiplication relationship with an array or area model and explain what the rows, columns, and product represent. Arrays and area models can help students represent equal groups, compose and decompose quantities, and examine the distributive property (Figure 1). Teachers can connect these representations to related ideas in fractions, algebra, and proportional reasoning. Research on representations suggests that connecting visual and symbolic forms can support learning when students attend to the mathematical relationships they represent (Fyfe et al., 2014). Comparing two representations, such as an array and a repeated-addition expression, can help students examine how each shows the same relationship.

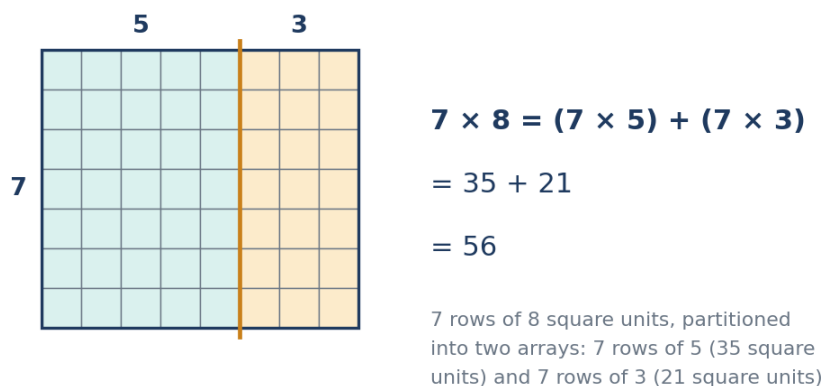


Figure 1. A 7×8 array partitioned into a 7×5 array (35 square units) and a 7×3 array (21 square units).

Key Idea • A word problem or a model is not automatically DOK 2. The level depends on the reasoning students must do and how familiar the task is.

DOK 3: Strategic Thinking and Justification

DOK 3 involves strategic reasoning and mathematical justification. Students may be asked to investigate a relationship, develop a claim, and use mathematical evidence to justify their conclusion. For example, students might investigate whether reversing the factors always produces an equal product, use representations to support their reasoning, and explain why the relationship holds beyond the examples they tested (Figure 2). Comparing two strategies for 36×25 and determining which is more efficient, and why, can also require strategic reasoning. Asking students to describe a familiar array may remain DOK 2.

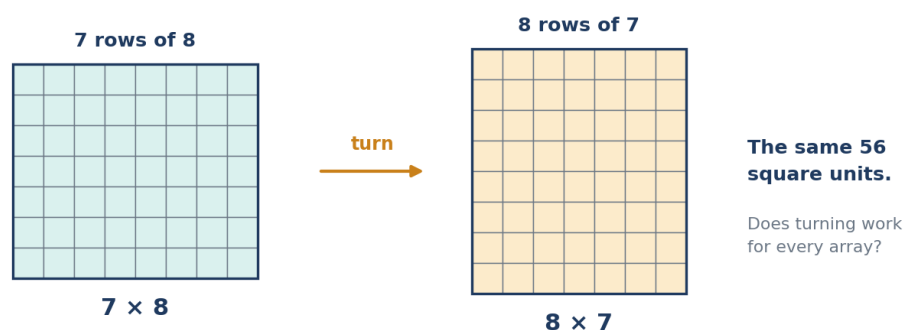


Figure 2. Turning a 7×8 array shows 8×7 . A general justification explains why this holds for every array.

Explaining and justifying mathematical relationships can help students connect procedures, representations, and underlying properties (Skemp, 1976). Research on self-explanation suggests that explaining relationships and worked examples can support learning (Chi et al., 1989); that study examined worked examples in physics, not DOK classifications. In mathematics, the reasoning students develop matters more than simply requiring an explanation. A question such as “How do you know the order does not change the product?” invites this reasoning, but the DOK level depends on what students must do in response.

DOK 4: Extended Reasoning Across Conditions

DOK 4 involves extended investigation in which students plan an approach, examine related cases, develop conjectures, and justify their conclusions. Students investigate rectangles with whole-number side lengths and a fixed area of 48 square units. They develop a systematic way to generate and organize possible rectangles, compare the resulting perimeters, formulate a conjecture about the relationship between side lengths and perimeter, and examine

whether the conjecture holds for other fixed areas. Students then use mathematical reasoning to support or revise their conclusions.

Dimensions	Area	Perimeter
1×48	48 square units	98 units
2×24	48 square units	52 units
3×16	48 square units	38 units
4×12	48 square units	32 units
6×8	48 square units	28 units

Depending on how the investigation is structured and what students are required to do, this task may involve DOK 3 or DOK 4. The time required to complete a task does not, by itself, determine whether it is DOK 4. Planning an investigation, organizing cases, and generalizing across conditions draw on the strategic competence and adaptive reasoning described in Adding It Up (National Research Council, 2001).

Implications for Instruction

Mathematical models can help students identify units, represent quantities, and examine relationships. Arrays, area models, and equations may represent the same multiplication relationship in different ways. Through explicit instruction, discussion, feedback, and practice, students can connect representations and develop accuracy, efficiency, and flexibility with mathematical procedures.

Questions can extend the reasoning required by a task, but the DOK level depends on what students must actually do in response. For example, after a student computes 6×4 , a teacher might ask, “Show this using an area model,” or “If you forgot that fact, how could you find it from a fact you know?” A student might respond by decomposing a factor: $6 \times 4 = (5 \times 4) + (1 \times 4) = 20 + 4 = 24$.

Clarifying Common Misconceptions About DOK

Several misconceptions about DOK persist. Higher DOK does not mean larger numbers or longer problems. A word problem is not automatically DOK 2 or DOK 3. Its classification depends on the mathematical reasoning required. DOK is not a staircase students must climb step by step; it classifies a task's demands (Webb, 2002). A task is also not permanently at a particular level: a task that asks for an explanation may become routine if students have already practiced that exact explanation. A higher DOK level is not automatically a better task. Instruction should include appropriate opportunities for recall, procedural practice, application, representation, and mathematical justification.

Key Idea • Recall, procedural practice, application, representation, and justification each have a purpose. Classify a task by what students must actually do.

Summary

Depth of Knowledge provides a framework for examining the cognitive complexity of mathematical tasks. It helps educators distinguish a task's difficulty from the reasoning required to complete it.

Effective mathematics instruction gives students opportunities to develop fluency with facts and procedures, interpret quantities, represent relationships, and justify mathematical claims. These forms of mathematical work complement one another and contribute to students' conceptual understanding, procedural fluency, and strategic competence (National Research Council, 2001).

DOK is most useful when educators examine the mathematical work students actually do, rather than assigning a level based on a task's appearance.

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Is a harder math problem always a deeper one?

Depth of Knowledge describes the reasoning a task requires, not how difficult it feels or how long it takes. Inside this DMT Insight, we look at:

- Why a page of larger multiplication problems can still be DOK 1
- When a word problem or an array is DOK 2, and when it may be routine
- How justifying why reversing the factors never changes the product becomes DOK 3 reasoning
- What an investigation must require to reach DOK 4

Read the full Insight at mathsuccess.io/dmtinsights.

What kind of thinking did your last lesson actually require?

