

DMT INSIGHTS

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SUMMER MATHEMATICS: DEVELOPING UNDERSTANDING, LANGUAGE, AND FLUENCY

What research suggests about student achievement, mathematical language, and summer instruction

Introduction

Summer gives students additional time to strengthen their mathematical knowledge. Summer instruction gives students opportunities to revisit mathematical relationships they are still developing while strengthening accuracy, efficiency, and flexibility with facts and procedures. Conceptual understanding and procedural fluency develop together, and both benefit from explicit instruction, feedback, and practice. Early mathematics knowledge is also among the strongest predictors of later achievement in both mathematics and reading (Duncan et al., 2007).

Mathematical language matters as well. Students need language to identify what a number represents, describe how quantities are composed or decomposed, and explain why a strategy works. This DMT Insight examines what research shows about the effects of summer mathematics programs on achievement, what it suggests about instruction that connects understanding and fluency, and how story-based mathematical language instruction may support both mathematics and language learning.

Evidence of Student Achievement

Research indicates that summer mathematics programs can improve student achievement. Lynch, An, and Mancenido (2023) conducted a meta-analysis of 37 experimental and quasi-experimental studies involving students from prekindergarten through Grade 12. Across these studies, summer programs that included mathematics produced an average positive effect of 0.10 standard deviations on mathematics achievement. The researchers found similar average effects in higher- and lower-poverty settings, and a secondary analysis found positive average effects on noncognitive outcomes.

Some individual programs have reported larger effects on the outcomes they measured. In a randomized study of the Elevate Math summer program, students entering Grade 8 participated in 19 days of mathematics instruction (Snipes et al., 2015). Students offered the program scored 0.70 standard deviations higher on an algebra-readiness assessment than students in the comparison group, and 29% met the specified algebra-readiness threshold, compared with 12% of the comparison group. Most participants had still not reached that threshold, and the study found no significant differences in students' interest in mathematics or self-efficacy.

These studies involve different students and outcomes, so their effect sizes are not directly comparable, and neither study evaluated DMTI's program. Results also vary across programs. Lynch and Kim (2017) found that students offered an online summer mathematics program with a laptop reported more mathematics engagement at home. However, treatment and comparison students performed similarly on achievement measures. Taken together, these findings suggest that summer mathematics programs can provide measurable academic benefits, and that the instruction students receive matters.

Key Idea • Research shows that summer mathematics programs can improve achievement, although the effects vary across programs, students, and assessments.

Developing Mathematical Understanding and Fluency

Summer instruction provides time to address mathematical relationships that students may not yet understand while also strengthening accuracy and efficiency with facts and procedures. Conceptual understanding and procedural fluency develop together. Rittle-Johnson, Siegler, and Alibali (2001) found that the two kinds of knowledge supported each other as students learned about decimals. Students need to understand the quantities and relationships a procedure represents, learn to carry it out accurately, and practice using it efficiently and flexibly.

In DMTI's approach, teachers use story mats, manipulatives, bar models, number lines, arrays, and equations to make mathematical relationships explicit. These representations are connected through mathematical language, teacher explanations, student discussion, and practice. For example, when students solve a multiplication problem, they can identify the unit being iterated, represent the equal groups, compose or decompose the product, and connect these relationships to multiplication equations and efficient procedures.

Research on summer programs demonstrates that additional mathematics instruction can improve achievement. Research on mathematics learning also suggests that conceptual understanding and procedural fluency support one another. Summer instruction can draw on both findings by connecting mathematical relationships with explicit instruction and practice.

Mathematical Language, Stories, and Student Learning

Mathematical language is closely connected to students' understanding of quantities and relationships. Students need language to identify what a number represents, explain how quantities are composed or decomposed, and describe why a mathematical strategy works.

Research on mathematical language supports the value of connecting stories and mathematical instruction. Hassinger-Das, Jordan, and Dyson (2015) randomly assigned 124 kindergarten students with early numeracy difficulties to a storybook intervention, a number sense intervention, or a comparison group. Students in the storybook intervention performed better on measures of mathematical vocabulary than students in the other conditions. The intervention did not produce corresponding gains on general mathematics measures.

Purpura, Napoli, Wehrspann, and Gold (2017) examined a mathematical-language storybook intervention with preschool students. Children who heard and discussed books rich in quantitative and spatial language outperformed the comparison group on measures of mathematical language and early numeracy, even though counting and arithmetic were not taught directly.

These studies measured mathematical vocabulary and early numeracy. They did not measure reading comprehension or evaluate DMTI story mats or summer programs. They do provide research support for the instructional rationale behind using stories to develop mathematical language.

A story mat provides a context in which students can identify quantities, represent relationships, and explain their reasoning. DMTI's farm story mat (Figure 1) provides a context for comparing quantities. A teacher might ask, "There are 12 hens and 5 pigs. How many more hens are there than pigs?" Students identify the two quantities, represent them with a bar model, and determine that there are 7 more hens than pigs (Figure 2). Students might decompose 5 into 2 and 3, subtract 2 from 12 to make 10, and then subtract 3 more. They can also explain the comparison using $5 + 7 = 12$.



Figure 1. DMTI's farm story mat.

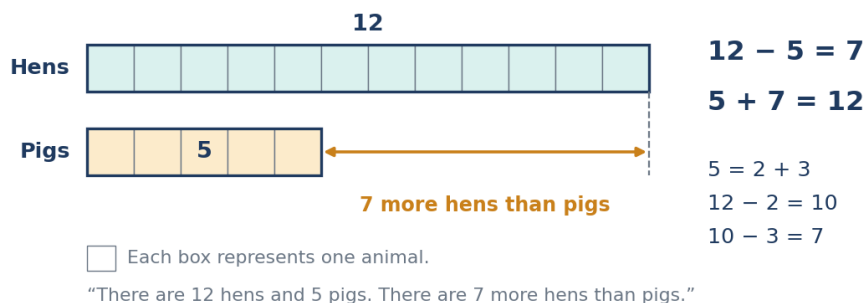


Figure 2. A bar model comparing 12 hens and 5 pigs. Each box represents one animal: $12 - 5 = 7$.

The mathematical language is connected to the quantities and operations students are learning. Teachers can explicitly introduce terms such as unit, compose, decompose, iterate, partition, and equal, and ask students to use those terms while explaining their representations and strategies. Determining whether DMTI's story mats also improve general language or reading scores requires evidence from assessments of those outcomes.

Key Idea • Research supports story-based instruction for developing mathematical language. Mathematical vocabulary, general language, reading, and mathematics achievement are distinct outcomes and should be measured separately.

Engagement and Participation

Engagement is another consideration in summer program design. Summer instruction can give students time to solve problems, discuss strategies, receive feedback, and practice the mathematics they are learning. Research suggests that some summer programs can improve students' participation and attitudes toward mathematics (Lynch et al., 2023). However, greater engagement does not necessarily produce higher assessment scores (Lynch & Kim, 2017). Programs should examine both how students participate in instruction and what mathematical knowledge they develop.

Implications for Summer Program Design

This research informs DMTI's approach to summer instruction. Story mats and contextual problems give students situations in which to identify quantities and units. Manipulatives, bar models, number lines, and arrays represent the relationships, and equations record them. Teachers explicitly introduce mathematical language, provide feedback, and use varied practice to build accuracy and efficiency, while students explain and justify their reasoning. Teachers select these representations to fit the mathematics; they do not follow a fixed sequence, and procedural practice remains an important part of instruction.

Summary

Research indicates that summer mathematics programs can improve student achievement, although outcomes vary. Research on story-based mathematical language instruction also suggests benefits for mathematical vocabulary and, in some studies, early numeracy.

DMTI's summer instruction connects mathematical language, representations, explicit instruction, and practice. Students identify units and quantities, compose and decompose numbers, represent relationships, and develop accurate and efficient procedures. Examining student responses and measuring mathematics and language outcomes can help teachers and program leaders evaluate what students are learning.

References

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Social Media

What can students gain from a summer mathematics program?

Research shows that summer programs that include mathematics can raise achievement, and that story-based mathematical language instruction can strengthen mathematical vocabulary. Inside this DMT Insight, we look at:

- What a meta-analysis of 37 studies found about summer mathematics achievement
- Why conceptual understanding and procedural fluency develop together
- What storybook studies show about mathematical language
- How a farm story mat and a bar model help students compare 12 hens and 5 pigs

Read the full Insight at mathsuccess.io/dmtinsights.

What mathematical language do your students use when they explain their thinking?

