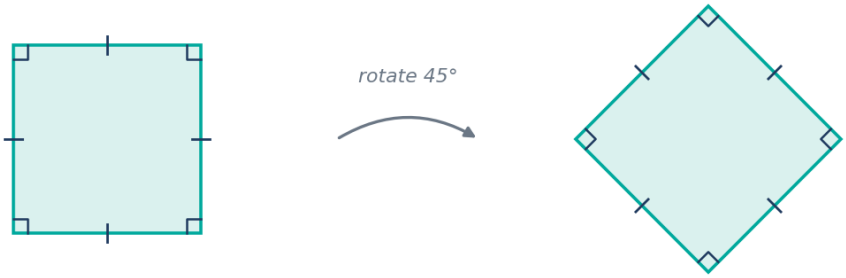
DMT INSIGHTS

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THE DIAMOND PROBLEM: HOW EVERYDAY LANGUAGE INFLUENCES GEOMETRIC THINKING

Introduction

A student points to a square that has been rotated and says, “That is a diamond.” Another student identifies the same figure as a square when it rests on a horizontal side. What accounts for the difference?



Square
4 congruent sides, 4 right angles

The same square, rotated
4 congruent sides, 4 right angles

What changed? Its orientation. What stayed the same? Every defining property.

Figure 1. Both figures are squares. Rotation changes the orientation, not the defining properties.

Students develop their understanding of shapes through the examples, language, and representations they encounter. When a square is called a “diamond” simply because it has been rotated, students may begin to associate orientation with a shape's name rather than examine its defining properties. In mathematics, a diamond is not a separately defined category of polygon. The figure is a square if it has four congruent sides and four right angles, regardless of its orientation. This distinction matters in schools because the mathematical terms we use should help students identify and reason about the properties that define shapes.

Research on geometric thinking, concept images, and classification helps explain how students develop these ideas and why familiar examples and everyday language can influence their reasoning.

From Recognizing Shapes to Reasoning About Properties

Early geometry gives students opportunities to move beyond recognizing familiar shapes and begin reasoning about the properties that define them. A student who recognizes a square by its appearance and a student who can explain that it has four congruent sides and four right angles are using different kinds of knowledge.

Duval (2006) examined the role of representations in mathematical understanding and the importance of distinguishing mathematical objects from their representations. A drawing of a square is one representation of that shape; its orientation does not determine its mathematical properties. In a study of young children, Verdine et al. (2016) found that children identified typical shape examples more accurately than atypical ones and knew less about the properties that define shapes than about their names.

A student's response to a shape therefore reflects several kinds of knowledge at once: the properties that define the category, the examples the student has encountered, and the everyday language used to name shapes.

Understanding Polygons Through Sides, Vertices, and Relationships

A polygon is a closed plane figure formed by straight line segments that meet at their endpoints. Polygons are classified by the number of sides and by additional properties, including side lengths, angle measures, and relationships among sides. These properties determine a figure's mathematical classification, not its orientation or familiar appearance.

Battista (2007) reviewed research on the development of geometric and spatial thinking and described how students' reasoning changes as they attend to the components of shapes and the relationships among them, rather than to shapes as visual wholes.

Students develop geometric reasoning as they examine the sides, vertices, angles, and relationships that define shapes. They begin to recognize that these properties remain the same when a figure is rotated, reflected, or moved to a new position. Enlarging or reducing a figure changes its side lengths but not its angle measures or parallel sides, so an enlarged square is still a square.

Definitions, Categories, and Hierarchies

The Common Core State Standards ask students to understand that shapes in different categories may share attributes and that shared attributes can define a larger category (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], 2010).

Orientation is not among these properties. Rotating, reflecting, or translating a figure does not change its side lengths, angle measures, or parallel relationships, so it does not change the figure's category.

A parallelogram is a quadrilateral with two pairs of parallel sides. Rectangles are parallelograms with four right angles, and rhombi are parallelograms with four congruent sides. Squares satisfy both sets of properties and therefore belong to both categories. Fujita and Jones (2007) found that learners who could state quadrilateral properties often did not classify them hierarchically and that prototypical images influenced their classifications. DMT Insight 46 examines quadrilateral definitions and hierarchical classification in detail.

Why Students Rely on Familiar Examples

Tall and Vinner (1981) distinguished a student's concept image, the total cognitive structure associated with a concept, including mental pictures, examples, and properties, from the concept definition, the words used to specify the concept. Vinner (1991) explained that students may rely on their concept images when classifying mathematical objects, even when they can state the formal definition. Duval's (2006) work on representations is related but distinct: it concerns how students coordinate figures, language, and symbols.

Research on young children's shape concepts describes the role of prototypes. In a study of 82 five- and six-year-olds, most children identified a prototypical triangle, but about half had difficulty identifying triangles of other sizes, types, and orientations; the authors concluded that the children relied on a visual prototype (Dağlı & Halat, 2016). Clements et al. (1999) found that preschool children often judged shapes by their visual qualities, while some also attended to properties such as the number of sides.

What Research Reveals About Geometric Classification

Prototypes and Orientation

Research with young children indicates that orientation and unfamiliar proportions can affect whether students recognize a figure as belonging to a category (Clements et al., 1999; Dağlı & Halat, 2016). These findings illustrate why recognition based on familiar examples is not the same as classification based on defining properties.

Category Inclusion

Students often treat shape names as mutually exclusive, so that a square cannot also be a rectangle. Hierarchical classification requires reasoning from definitions: a square has every defining property of a rectangle, so it is a rectangle (Fujita & Jones, 2007).

Why Mathematical Language Matters

The word "diamond" commonly describes a figure with a vertex pointing upward. However, it is not a separately defined mathematical category of polygon. A square does not become a different shape when it is rotated, and a rhombus does not require a particular orientation.

When classroom materials identify a "diamond" alongside triangles, squares, and rectangles, they can reinforce the idea that orientation determines a shape's name. Teachers should use the correct mathematical terms and help students examine the properties that define each figure.

Research on concept images helps explain why this matters. The words and examples students encounter influence how they recognize and classify shapes (Tall & Vinner, 1981; Vinner, 1991).

The Influence of Regular and Familiar Shapes

Repeated exposure to regular, symmetric examples may shape how students classify polygons. Prototype research suggests that students may struggle to recognize less familiar examples, even when those figures meet the category's defining properties (Clements et al., 1999; Dağlı & Halat, 2016).

The Development of Geometric Reasoning

Van Hiele (1986) described different levels of geometric reasoning, including recognizing figures by appearance, examining their properties, and reasoning about relationships among properties and categories. These levels help explain differences in how students justify their classifications. Students' experiences and instruction, rather than age alone, influence progress in geometric reasoning.

DMTI emphasizes mathematical language and reasoning about properties and relationships. Students need opportunities to compose and decompose shapes, compare examples and nonexamples, and explain why a figure belongs to a particular category. The emphasis is on understanding the properties that define shapes rather than memorizing names from familiar pictures.

Conclusion

Research on geometric thinking helps explain why students often rely on familiar examples and everyday language when identifying and classifying shapes. The images and terms students encounter influence what they recognize and which properties they attend to.

The diamond problem illustrates why mathematical language matters. A square remains a square when rotated because its defining properties have not changed. Calling it a "diamond" may describe its appearance, but it does not identify a different mathematical shape.

In schools, we should consistently use the correct mathematical names and help students examine the sides, vertices, angles, and relationships that define figures. Students should be able to explain why a shape belongs to a category, including why a square is also a rectangle and a rhombus.

The goal is for students to reason from mathematical properties and relationships rather than relying on orientation or familiar appearance.

References

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843–908). Information Age Publishing.

Clements, D. H., Swaminathan, S., Hannibal, M. A. Z., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192–212.

Dağlı, Ü. Y., & Halat, E. (2016). Young children's conceptual understanding of triangles. *Eurasia Journal of Mathematics, Science & Technology Education, 12*(2), 189–202.

Duval, R. (2006). A cognitive analysis of problems of comprehension in the learning of mathematics. *Educational Studies in Mathematics, 61*(1–2), 103–131.

Fujita, T., & Jones, K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education, 9*(1–2), 3–20.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Authors.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2), 151–169.

van Hiele, P. M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.

Verdine, B. N., Lucca, K. R., Golinkoff, R. M., Hirsh-Pasek, K., & Newcombe, N. S. (2016). The shape of things: The origin of young children's knowledge of the names and properties of geometric forms. *Journal of Cognition and Development, 17*(1), 142–161.

Vinner, S. (1991). The role of definitions in the teaching and learning of mathematics. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 65–81). Kluwer.

Social Media

A student calls a rotated square a "diamond." Another says a square cannot be a rectangle because rectangles are longer.

"Diamond" is not a separate mathematical shape. A rotated square is still a square. Research on prototypes and concept images helps explain why students may classify shapes by familiar appearance rather than by defining properties.

In DMT Insight 41, we review research on how examples, language, and orientation influence the categories students form, and how geometric reasoning develops.

Read the full Insight at mathsuccess.io/dmtinsights.

Which shape do your students most often classify by appearance?

