DMT INSIGHTS

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Math Success by DMTI

DRAWING UNDERSTANDING: WHY STUDENTS SHOULD BUILD THE NUMBER LINE

Introduction

A student places 47 on a number line printed from 0 to 100 and marks a point a little past the middle. The placement is correct. Ask that student what one unit on the line represents, or how far 47 is from 0, and the reasoning often does not follow. The printed line has already fixed the endpoints, the spacing, and the position of 0, so the student reads a location instead of measuring a distance. Estimation on a bounded, pre-marked line behaves as a proportion judgment about the printed interval rather than as a report of numerical magnitude (Barth & Paladino, 2011), and performance on bounded tasks reflects the measurement strategies that the printed frame invites (Cohen & Sarnecka, 2014).

Constructing the line returns those decisions to the student. Choosing the endpoints, defining a unit, iterating that unit, and partitioning the intervals are the mathematical actions that the printed line performs on the student's behalf. Those decisions matter beyond the single task. Number-line estimation is strongly associated with mathematical competence across ages and measures (Schneider et al., 2018), and linear representations of number improve young children's numerical understanding under experimental conditions (Siegler & Ramani, 2009). The difficulty is equally well documented. Students hold rigid conceptions of the line, expecting 0 at the center or increments of 1, and their understanding develops through identifiable stages rather than arriving all at once (Ünal et al., 2024). This Insight examines what students do when they construct a number line, why those actions support measurement, fractions, and graphing, and how teachers can build construction into instruction and professional learning.

> **Key Idea • A correct placement on a printed line can hide the reasoning it appears to show. When students choose the endpoints, define the unit, iterate it, and partition the intervals, they perform the measurement that the printed line performs for them.**

The Structures Students Build: Unit, Partition, and Scale

Constructing a number line requires students to decide what one unit represents and then to hold that decision constant across the line. Defining a unit length and iterating it produces the equal-sized intervals that make a position readable, and partitioning those intervals produces the smaller units that fractions and decimals require. These are measurement actions, and students use the same actions with a ruler, a fraction strip, and the axis of a graph. On a printed line the equal spacing is already true, so students can locate a number correctly without ever having established that equality themselves. Bounded, pre-marked tasks reward proportion strategies anchored to the printed endpoints, and performance on them develops differently from performance on unbounded tasks, where no frame is available to reason from (Cohen & Sarnecka, 2014).

Mapping number to space connects the line to a wider body of evidence. Spatial skill is associated with early number knowledge, and the linear number line accounts for part of that relationship (Gunderson et al., 2012). Number-line estimation is in turn strongly associated with broader mathematical competence (Schneider et al., 2018). Students who never construct a line often carry rigid expectations into later work: that 0 belongs at the center, that each tick advances by 1, and that the line stops where the paper stops. These expectations are documented features of student thinking, and they persist into high school (Ünal et al., 2024). Constructing lines with different endpoints and different units gives students a reason to revise them.

> **Key Idea • Building a line is measurement. Students define a unit, iterate it to produce equal-sized intervals, and partition those intervals, and the equality that makes a position readable becomes something they establish rather than something the page provides.**

Measurement and Fractions: Number as Length

Locating a number on a line that students construct themselves requires them to coordinate two systems at once, the count of units and the length that those units cover. Fifth graders working on open number lines coordinate numerical and linear units in identifiable ways, and their placements shift with the endpoints and the reference points available to them, which shows how much of the reasoning the printed frame ordinarily carries (Saxe et al., 2013). Partitioning the same line into fourths and then into tenths lets students treat 1/4 and 1/10 as distances measured from 0 rather than as points to be recalled, and comparison, equivalence, and scaling follow from that partition.

Physical action strengthens that connection. Training in which students walk a number line, stepping off units with their own bodies, improves number-line performance and carries over to related numerical tasks (Link et al., 2013). Constructing a line on paper is a smaller version of the same action. The hand marks the unit, the eye checks the spacing, and the language names what the marks represent. Dual coding theory proposes that the verbal and the visual systems are separate and interconnected, so a model that students both build and describe is encoded in both (Paivio, 1986).

> **Key Idea • When students treat the unit as a length, a fraction becomes a distance measured from 0 rather than a point to be recalled, and comparison, equivalence, and scaling follow from the partition.**

From the Number Line to Data and Graphing

A line plot is a number line with data placed on it, and an axis is a scaled number line. When students invent their own displays, they confront the decisions that a prepared grid settles in advance: what range to show, how far apart to place the marks, and what each interval represents. Students who design representations of motion negotiate exactly these questions, and the representational decisions become the mathematical content of the lesson (diSessa et al., 1991). Classrooms in which students construct displays to model natural variation develop conceptions of distribution that prepared displays do not require them to form (Lehrer & Schauble, 2004).

Coordinate graphing rests on the same structure. The x-axis is a scaled number line, and a student who has established equal-sized units on a line already holds the reasoning that origin placement and interval choice require. Equal spacing represents equal units on a bar graph for the same reason that it does on a ruler. However, when the grid arrives pre-formatted, students plot points accurately while the scale remains someone else's decision.

> **Key Idea • An axis is a scaled number line. Students who set the range, the interval, and the origin themselves carry that reasoning into line plots, bar graphs, and the coordinate plane.**

Instructional Design: Turning Research into Practice

Routines that begin with a blank line give students the decisions worth making. A segment with no ticks and no labels requires students to establish where 0 belongs, what one unit represents, and how to check that the spacing is equal. Varying the endpoints across tasks, from 0 to 1, from 0 to 50, and from 2 to 10, prevents students from reusing a single unit and requires them to rescale, which works directly against the rigid conceptions that the research documents (Ünal et al., 2024).

Fraction and measurement contexts give the construction something to measure. Asking students to partition a meterstick line into sixths, or to place 3/4 and 5/8 on a line they have drawn, connects the physical length to the visual model and to the symbols. Drawing the axes before graphing does the same work for data. Reflection prompts such as How did you choose your unit? and How do you know that your spacing is equal? make the structural reasoning audible. Student-constructed lines also provide assessment evidence that an answer key does not, because the spacing, the labels, and the placement of 0 show how a student is thinking about the unit rather than only whether a point landed in the correct place.

> **Key Idea • Routines that start with a blank line, vary the endpoints, and ask students to justify the unit make structural reasoning visible and give teachers evidence about thinking rather than only about placement.**

Professional Learning: A Shared Language for Structure

Teachers who construct number lines together experience the decisions their students face. Choosing a unit for a line from 0 to 1 and then for a line from 0 to 50 makes the demands of rescaling concrete, and analyzing student-constructed lines shows how much the spacing and the labels reveal. Consistent structural language supports that work. When teachers use unit, compose, iterate, decompose, partition, and equal in the same way across grades, classroom discourse names mathematical actions rather than procedures (Brendefur & Strother, 2021). Connecting number-line work to measurement, to fractions, and to graphing lets teachers treat the line as one model that carries through the K–8 curriculum rather than as a topic that belongs to a single unit.

> **Key Idea • When teachers build and analyze number lines together, using one structural vocabulary across grades, they can design tasks that target the reasoning behind a placement rather than the placement alone.**

Conclusion

Number-line estimation is strongly associated with mathematical competence (Schneider et al., 2018), linear representations of number improve young children's understanding under experimental conditions (Siegler & Ramani, 2009), and the coordination of numerical and linear units is difficult enough that students' placements shift with the frame that they are given (Saxe et al., 2013). Together these findings point to one instructional move. A line that students construct requires them to establish the unit, the partition, and the scale that a printed line establishes for them, and those three decisions carry forward into measurement, fractions, data displays, and the coordinate plane. The number line then ceases to be a display that students read. It becomes a model that students construct.

> **Key Idea • A constructed line requires students to establish the unit, the partition, and the scale that a printed line provides for them, and those decisions carry forward into measurement, fractions, data displays, and the coordinate plane.**

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Social Media

What does a correct placement on a number line actually show?

A student marks 47 on a line printed from 0 to 100 and lands in the correct place. Ask how far 47 is from 0, or what one unit on that line represents, and the answer often does not come. The printed line has already fixed the endpoints, the spacing, and the position of 0, so the student reads a location rather than measuring a distance. When students construct the line themselves, they choose the endpoints, define a unit, iterate it, and partition the intervals, which are the measurement actions the printed line performs for them. This Insight examines what students do when they construct a number line and why those actions support measurement, fractions, and graphing.

Inside this Insight, you will learn:

- Why a correct placement can hide the reasoning behind it
- What students establish when they choose the unit, the partition, and the scale
- How construction connects number to length, and length to fractions and measurement
- Why an axis is a scaled number line

Read the full Insight at mathsuccess.io/dmtinsights.

When your students use a number line this week, are they reading one you built for them, or building one themselves?

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