

DMT INSIGHTS

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WHY TEACHER KNOWLEDGE MATTERS: THE RESEARCH BEHIND EFFECTIVE MATHEMATICS INSTRUCTION

Introduction

Over the past several decades, schools have invested considerable resources in mathematics standards, assessments, and curriculum materials. However, student achievement remains a concern, particularly in elementary mathematics. On the 2024 National Assessment of Educational Progress, 39 percent of fourth graders scored at or above the Proficient level in mathematics. The average fourth-grade score was 2 points higher than in 2022 and 3 points lower than in 2019 (National Center for Education Statistics [NCES], 2025). This raises an important question: Why have these investments not consistently produced the improvements educators have sought?

One area that deserves greater attention is teachers' mathematical and instructional knowledge. Research has examined the specialized mathematics teachers need, how their knowledge relates to student achievement, and which forms of professional learning support changes in instruction.

This DMT Insight reviews that research and considers what it suggests about teacher preparation, professional development, and the balance between investing in curriculum materials and investing in the teachers who use them.

Mathematical Knowledge for Teaching

Mathematical knowledge for teaching differs from the general mathematical knowledge a well-educated adult uses. Ball et al. (2008) described several domains of this knowledge. Common content knowledge is the mathematics used to solve problems. Specialized content knowledge is the mathematics teachers use to analyze student errors, evaluate unfamiliar strategies, choose representations, and explain why procedures work. Knowledge of content and students combines mathematics with knowledge of how students are likely to reason about it, and knowledge of content and teaching combines mathematics with decisions about sequencing examples and selecting representations.

For example, any adult can calculate $43 - 29$. A teacher must also determine whether a student's strategy of counting up from 29 is valid, explain why decomposing a ten does not change the quantity, and recognize what an answer of 26 suggests about how the student is treating the digits.

Hill et al. (2005) examined whether this knowledge is related to student learning. They measured teachers' mathematical knowledge for teaching with items that asked teachers to analyze student work and explain mathematical ideas. They followed students in first- and third-grade classrooms over a school year. After controlling

for student and teacher characteristics, teachers' scores were significantly related to students' achievement gains. These findings are correlational. They show that teacher knowledge and student learning are related, but they do not show that a particular course or professional development program will produce gains.

Key Idea • Knowing how to solve a problem differs from knowing how to teach it. Mathematical knowledge for teaching includes interpreting student strategies and explaining why procedures work.

Mathematical Structure and Teacher Knowledge

Understanding mathematics for teaching involves recognizing the relationships that connect mathematical ideas. Concepts such as unit, quantity, equivalence, composition, decomposition, iteration, and partitioning appear throughout elementary mathematics. Teachers need to understand these relationships and how they develop across grade levels.

Ma (1999) compared U.S. and Chinese elementary teachers' explanations of subtraction with regrouping, multidigit multiplication, division by fractions, and the relationship between perimeter and area. Many Chinese teachers explained procedures through underlying relationships, such as decomposing a higher-value unit, while many U.S. teachers described the steps of the procedure. Ma described the knowledge of the strongest teachers as a profound understanding of fundamental mathematics: connected, coherent, and organized around a small number of basic ideas.

Research on whole-number addition and subtraction shows that children's understanding of multidigit numbers depends on relating number words, written numerals, and quantities grouped in tens (Fuson, 1992). The Mathematical Education of Teachers II recommends that prospective elementary teachers study the mathematics of the elementary grades in depth, from a teacher's perspective, in at least 12 semester hours of coursework (Conference Board of the Mathematical Sciences [CBMS], 2012).

DMTI organizes instruction around these structural ideas, using a consistent language of unit, compose, decompose, iterate, partition, and equal. That language is an instructional framework informed by this research. The studies cited here do not prescribe it.

How Students Learn Mathematics

Teachers also need knowledge of how students develop mathematical ideas. Clements and Sarama (2014) describe learning trajectories that combine a mathematical goal, a developmental progression of children's thinking, and instructional activities matched to each level. A trajectory helps a teacher anticipate how a student's thinking is likely to develop and what may come next.

Spatial reasoning is related to mathematics learning. Mix and Cheng (2012) reviewed evidence that spatial skills and mathematics performance are associated across development and considered whether spatial training might support mathematics, a question the research had not yet settled.

Multi-step mathematics demands working memory, and individual differences in working memory relate to mathematics performance (Raghubar et al., 2010). Paivio's (1986) dual-coding theory proposes that information

represented both verbally and visually is more readily remembered. For teachers, these findings point to how many steps students must hold in mind and how models and language can represent the same relationship.

Rittle-Johnson and Alibali (1999) found that instruction on the concept of mathematical equivalence led fourth- and fifth-graders to generate correct procedures that transferred to related problems, while instruction on a procedure improved conceptual understanding but led to more limited transfer. A later review described the relationship as bidirectional: gains in one kind of knowledge support gains in the other (Rittle-Johnson et al., 2015). Conceptual understanding and procedural fluency are not competing instructional goals.

Key Idea • Conceptual understanding and procedural fluency develop together. Teachers need knowledge of how both develop.

Teacher Preparation: What the Research Identifies as Necessary

Professional organizations have described what teacher preparation should include. The Association of Mathematics Teacher Educators (AMTE, 2017) calls for candidates to develop knowledge of mathematics, of students as learners, and of teaching practices together. The CBMS (2012) recommends that teachers study elementary mathematics more deeply from a teacher’s perspective. Adding It Up describes mathematical proficiency as five intertwined strands, including conceptual understanding and procedural fluency, and emphasizes the knowledge teachers need to support them (National Research Council, 2001). These documents establish expectations. They do not show how often programs meet them.

Empirical evidence on what programs provide is more limited. In a review of 77 education schools, the National Council on Teacher Quality found that required mathematics coursework for elementary candidates ranged from zero to six courses, and 42 percent of the schools had no dedicated elementary mathematics methods course (Greenberg & Walsh, 2008). The review drew on syllabi, textbooks, and course requirements rather than classroom observations, but the range it documented shows how much preparation can vary.

Grossman et al. (2009) argue for organizing teacher education around core teaching practices that candidates learn by rehearsing and analyzing. In DMTI’s work with schools, many teachers describe having had limited opportunities to study elementary mathematics in depth or to analyze student work before entering the classroom. This is an observation from practice, not a research finding, but it aligns with the variation the research describes.

Consequences for Mathematics Instruction

Teacher knowledge is related to the mathematics students encounter during instruction. In a study of the mathematical quality of instruction, Hill et al. (2008) found a strong, positive association between teachers’ mathematical knowledge for teaching and the mathematical quality of their lessons. They identified factors, including curriculum materials and teachers’ beliefs, that supported or limited how teachers used their knowledge.

Lessons rated lower in mathematical quality often rely on rules disconnected from the mathematics, such as “cross-multiply” or “move the decimal,” or use models as steps to follow rather than representations of quantities and relationships. A rule can produce correct answers on familiar problems while leaving the relationship it records

unexamined. These examples illustrate the research; they do not show that every such lesson results from gaps in a teacher's preparation.

Access to knowledgeable teaching is not evenly distributed. In Washington State, students from low-income families and students of color were more likely than their peers to be taught by less experienced teachers (Goldhaber et al., 2015).

What Research Says About Professional Development

Garet et al. (2001) surveyed a national sample of more than 1,000 mathematics and science teachers. Teachers reported greater knowledge gains and more changes in practice when professional development focused on content, included active learning, and aligned with their other professional experiences. Longer activities and collective participation by teachers from the same school were associated with those features. The findings rest on teachers' reports.

Desimone (2009) proposed a core framework for studying professional development built on five features: content focus, active learning, coherence, duration, and collective participation. Yoon et al. (2007) reviewed studies of professional development and student achievement and found nine that met their evidence standards. In those studies, programs offering more than 14 hours of professional development showed positive effects on achievement.

Experimental results are mixed. A meta-analysis of 95 experimental and quasi-experimental studies of STEM professional development and curriculum programs found an average effect of 0.21 standard deviations on student outcomes across both types of programs. Programs that helped teachers learn to use curriculum materials, focused on content knowledge, pedagogical content knowledge, or how students learn, and included meetings to discuss classroom implementation had stronger outcomes (Lynch et al., 2019). In a randomized study, content-intensive professional development for fourth-grade teachers improved teachers' mathematical knowledge and some aspects of instruction without improving student achievement (Garet et al., 2016).

These studies have limitations. Many rely on teachers' self-reports, effects vary widely across programs, and features such as duration describe opportunities for learning rather than guarantee it. One- or two-day workshops that are disconnected from teachers' curriculum and classrooms offer few of the features the research identifies. That does not mean every short session is ineffective or that more hours alone produce improvement. What teachers study, and how that learning connects to their classrooms, matters.

Investing in Curriculum and Investing in Teachers

Curriculum materials are important, but their effectiveness depends in part on how teachers interpret and use them. Teachers must understand a lesson's mathematical ideas, anticipate student thinking, and decide how to connect representations, discussion, and procedures. Providing stronger curriculum materials without giving teachers enough opportunities to develop this knowledge leaves an important part of instructional improvement unaddressed.

Most of a district's budget pays for people: in 2020–21, salaries and employee benefits accounted for 79 percent of current expenditures in U.S. public elementary and secondary schools (NCES, n.d.). Instructional materials can be relatively inexpensive. One analysis found that some of the highest-quality elementary mathematics curricula cost

about \$36 per student (Boser et al., 2015). Teacher development spending is harder to track because it is spread across staff positions, teacher time, substitutes, and contracted services. TNTP (2015) estimated that three large districts spent about \$18,000 per teacher per year on teacher development across all subjects. That estimate is not specific to mathematics and should not be compared directly with curriculum purchase costs.

Curriculum alone has shown limited effects. In a study of about 6,000 schools in six states, schools using different fourth- and fifth-grade mathematics textbooks showed little difference in average achievement growth. Teachers reported little curriculum-specific professional development, averaging 0.8 to 1.6 days, depending on the textbook (Blazar et al., 2019). In contrast, Cohen and Hill (2001) found that California teachers who spent more time in workshops tied to their students' curriculum were more likely to report practices consistent with the state's mathematics framework, and schools where more teachers reported those practices tended to have higher test scores.

Curriculum and professional learning are not competing alternatives. The more useful question is how schools allocate resources to both, and whether teachers have enough opportunity to learn the mathematics and the instructional decisions their materials require.

Key Idea • Curriculum materials organize instruction. Teachers' knowledge determines how they use those materials. Investment in one should match investment in the other.

Conclusion

Research on mathematical knowledge for teaching, student learning, teacher preparation, and professional development points to the importance of developing teachers' mathematical and instructional knowledge. Curriculum materials matter, but teachers must understand the mathematics they are teaching and how students develop those ideas.

Improving mathematics achievement requires attention to both the resources schools adopt and the professional learning teachers receive. Investments in curriculum should be accompanied by sustained opportunities for teachers to develop the knowledge needed to use those materials effectively.

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Social Media

Schools have invested in standards, assessments, and curriculum materials. In 2024, 39 percent of fourth graders scored at or above Proficient on the NAEP mathematics assessment.

Research on mathematical knowledge for teaching shows that teachers’ specialized knowledge, such as interpreting student strategies and explaining why procedures work, is related to student achievement.

In DMT Insight 37, we review the research on mathematical knowledge for teaching, how students learn mathematics, teacher preparation, professional development, and the balance between investing in curriculum and investing in teachers.

Read the full Insight at mathsuccess.io/dmtinsights.

How does your district balance investments in curriculum materials and in teachers’ mathematical knowledge?



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