

DMT INSIGHTS

Issue 35 · 2026 · by Jonathan Brendefur, PhD and Associates



VARIED PRACTICE: CONNECTING MATHEMATICAL REPRESENTATIONS

How contexts, models, language, and symbols support mathematical understanding and fluency

Introduction

Mathematical practice serves several purposes. Students need opportunities to develop accuracy and efficiency with facts and procedures. They also need opportunities to examine the quantities and relationships those procedures represent.

Varied Practice connects these purposes. Students work with the same mathematical relationship through different representations: a context or story, an iconic model, and mathematical symbols or language. In DMTI's three-column format, students may be given one representation and asked to construct the others.

For example, a student might examine a bar model, write a corresponding contextual problem, and represent the relationship with an equation. The teacher can then ask students to explain how the quantities and relationships are represented in each form.

The purpose is to develop students' ability to recognize, represent, and use mathematical relationships. Conceptual understanding and procedural fluency develop together through instruction, explanation, feedback, and practice.

Theoretical Foundations

Bruner (1966) described three ways of representing ideas: enactive (through action), iconic (through images), and symbolic (through language and symbols). In mathematics, students might act out a situation with physical materials, draw a bar model or number line, and write an equation. These modes do not form a fixed sequence, and they do not correspond directly to the three columns of a Varied Practice task. A contextual problem is a story or situation, and students may represent it with materials, a drawing, words, or symbols. Bruner's framework is useful because it highlights how actions, images, and symbols can each show the same mathematical relationship.

Research on multiple representations suggests that students can benefit from examining mathematical ideas in different forms (Ainsworth, 2006; Lesh, Post, & Behr, 1987). Ainsworth described several functions of multiple representations: one representation can complement another, constrain how another is interpreted, or help learners build a deeper understanding by relating the two. She also noted that learners must understand each representation

and how the representations correspond, which places its own demands on students. Lesh, Post, and Behr described translating among representations as an important part of mathematical understanding.

For example, a first grader might use a bar model showing 14 red apples and 9 green apples to represent $14 + 9 = 23$. A fifth grader might identify 24 as the unit of 1, partition that quantity into four equal parts, and iterate one-fourth three times to represent three-fourths of 24, or 18. In each case, the teacher asks the student to explain what each part of the model and each number represents.

Key Idea • The same mathematical relationship can be represented with a context, a model, and symbols. The instructional value comes from students explaining how the representations correspond.

Dual Coding Theory: Words, Diagrams, and Symbols

Paivio's dual coding theory explains how verbal and nonverbal representations support learning and memory (Paivio, 1971, 1986). In mathematics, words, diagrams, and symbols can represent related information in different ways.

The instructional value depends on whether these representations help students examine the mathematics. A bar model may make a quantitative relationship visible, while an equation can express that relationship symbolically.

For example, the equation $24 \div 6 = 4$ can represent partitioning 24 objects into six equal groups, with four objects in each group. A corresponding diagram can help students identify what each number represents and explain the relationship between the quantities.

Additional representations are not automatically beneficial. Teachers need to select models and explanations that clarify the mathematics without introducing unnecessary complexity.

What Students' Representations Show

When students construct a representation, they show how they interpret the mathematics. The representations they produce, and their explanations, give teachers evidence to decide what to develop next.

Consider the equation $9 - 5 = 4$. One student writes, "I had 9 apples and gave away 5. Now I have 4." Another writes, "Tom has 9 apples, and Mary has 5. Tom has 4 more apples than Mary." Both contexts represent the equation, but they involve different subtraction relationships: removal and comparison (Figure 1). Asking students to construct and explain both situations shows how they interpret subtraction and whether they recognize the different relationships the same equation represents.

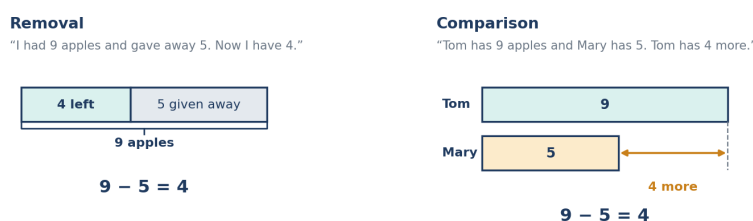


Figure 1. The same equation, $9 - 5 = 4$, represents a removal situation and a comparison situation.

When a student's bar model and equation do not represent the same relationship, the mismatch gives the teacher and student something specific to examine. Students do not always notice these inconsistencies on their own. Questions

such as “What does this part of your model represent?” help students compare the representations and revise their thinking.

Research on comparison supports a related practice. Rittle-Johnson and Star (2007) found that seventh-grade students who compared two solution methods for the same equation side by side made greater gains in procedural knowledge and flexibility than students who studied the methods one at a time. That study compared solution methods, not representations. It supports making comparison explicit; it does not evaluate the three-column format.

Varied Practice also differs from interleaved practice, in which different problem types are mixed within a practice set. Rohrer and Taylor (2007) found that college students who practiced mixed problem types performed better on a test one week later than students who practiced one type at a time. The two can be used together. A set of three-column tasks can include different mathematical structures, such as removal and comparison subtraction, so students must determine which relationship each situation represents.

Key Idea • A student’s representation is evidence of how the student interprets the mathematics. Two correct stories for $9 - 5 = 4$ can show different subtraction relationships.

Designing Varied Practice: The Three-Column Format

In DMTI’s three-column format, each row addresses one mathematical relationship. The columns are Contextual (Story or Situation), Iconic (Visual Model), and Symbolic (Equation and Mathematical Language). Figure 2 shows one row for a comparison problem.

Contextual (Story or Situation)	Iconic (Visual Model)	Symbolic (Equation and Mathematical Language)
There are 58 green apples in one basket and 36 red apples in another basket. How many more green apples are there than red apples?	 A bar model with two horizontal bars. The top bar is light blue and labeled '58 green'. The bottom bar is light orange and labeled '36 red'. A dashed vertical line extends from the end of the orange bar up to the blue bar. A double-headed orange arrow spans the length of the blue bar that is not covered by the orange bar, with the label 'difference: 22' below it.	$58 - 36 = 22$ “I compared 58 and 36. There are 22 more green apples than red apples.”

Figure 2. One relationship in the three columns: context, bar model, and equation with mathematical language.

The three-column format is a DMTI instructional application informed by research on multiple representations. The studies discussed here do not directly evaluate this particular format. Its value depends on the tasks teachers select and how they use students’ responses.

For initial instruction, a teacher may provide two representations and ask students to construct the third. As students become more familiar with the mathematical relationship, teachers may provide one representation and have students construct the remaining two. The task should also vary the mathematical structure, not simply change the format. Students should examine how units, quantities, and relationships are preserved across representations.

Teachers can vary which representation is provided. For example:

- **Equation given.** A teacher provides $8 + 4 = 12$. Students draw a bar model or number line and write a story the equation represents.
- **Model given.** Students are given a bar model and construct the corresponding story and equation.

- **Context given.** Students read a situation, represent it with a model, and write the equation.

Design principles include:

- **Consistency.** The same mathematical relationship should underlie all three columns.
- **Variation.** Vary which column is provided, and vary the mathematical structure, such as removal and comparison subtraction.
- **Explanation.** Ask students to identify the quantities, explain the units, and justify how each representation shows the relationship.
- **Intentional models.** Use bar models, number lines, area models, or other representations that make the relationship visible. There is no fixed order in which representations must be introduced.
- **Mathematical language.** The third column can ask students to describe the mathematics in words. For one-fourth, a student might write: “I partitioned the unit of 1 into four equal parts. Each part represents one-fourth of the unit of 1.”

Key Idea • Vary which representation is given and vary the mathematical structure. The goal is not more representations; it is understanding the relationship they show.

Summary

Varied Practice gives students opportunities to examine mathematical relationships through contexts, visual models, equations, and mathematical language.

When students construct and compare representations, teachers can ask them to identify the quantities, explain the units, and justify how the same relationship is represented in different ways.

These experiences complement explicit instruction and procedural practice. Together, they can help students develop mathematical understanding, accuracy, efficiency, and flexibility.

The important consideration is not how many representations students produce, but whether they understand and can use the mathematical relationships those representations show.

References

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Social Media

What does it mean for a student to solve an equation but not explain the quantities it represents?

Consider $58 - 36 = 22$.

A student may calculate the difference correctly. Can that student also create a comparison story, represent the quantities with a bar model, and explain how the model and equation show the same relationship?

DMTI's Varied Practice tasks give students opportunities to connect contexts, iconic models, mathematical language, and symbols.

The goal is to develop mathematical understanding and procedural fluency together.

Read the full DMT Insight at mathsuccess.io/dmtinsights.

What mathematical relationship do your students have difficulty representing in different ways?

