

DMT INSIGHTS

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UNDERSTANDING MISTAKES AND MISCONCEPTIONS: WHEN MATHEMATICAL IDEAS NO LONGER WORK

Introduction

A student says that 0.45 is greater than 0.7 because 45 is greater than 7. The student may be applying a familiar idea from comparing whole numbers. However, comparing decimals requires attention to place-value units. The 7 in 0.7 represents 7 tenths, while 0.45 represents 4 tenths and 5 hundredths. Understanding the student's reasoning helps the teacher determine which mathematical relationship needs further development.

DMT Insight 5 examines misconceptions, and Insight 23 considers student responses as assessment evidence. This Insight focuses on instruction: examine where a familiar idea applies, identify why it no longer works, and teach the mathematical relationship needed in the new situation.

Key Idea • Investigate the reasoning behind an error before deciding what to teach.

Distinguishing Mistakes from Misconceptions

A mistake may be a calculation or recording error, such as miscounting or misaligning digits. A misconception involves an idea or relationship that a student is applying incorrectly or too broadly. Misconceptions often extend knowledge that worked in an earlier situation. Smith, diSessa, and Roschelle (1993) argue that such conceptions are knowledge in transition: instruction refines and reorganizes them rather than simply replacing them.

A single incorrect response does not establish which is occurring. Asking the student to explain a strategy or represent the quantities provides the evidence needed to decide. A calculation error may need specific feedback and practice. A misconception may require the student to reconsider the units and relationships being used.

When a Familiar Mathematical Idea No Longer Applies

The three examples below show a student applying an idea from one situation to another where it no longer applies. Each response suggests a possibility to investigate, not a conclusion about the student.

Fractions and the unit of 1. A student says that $\frac{1}{8}$ is greater than $\frac{1}{4}$ because 8 is greater than 4. The student may be applying whole-number comparisons to the denominators. Ask the student to show both fractions with the same unit

of 1. Partitioning the unit of 1 into four equal parts and into eight equal parts shows that each eighth is half the size of each fourth (Figure 1). Partitioning the same unit of 1 into more equal parts produces a smaller fractional unit.

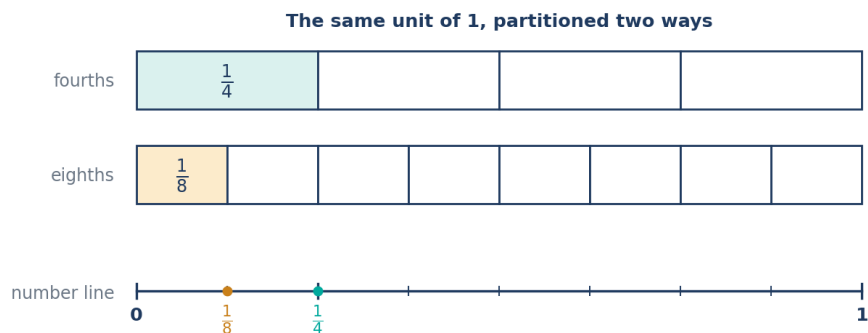


Figure 1. One fourth and one eighth of the same unit of 1.

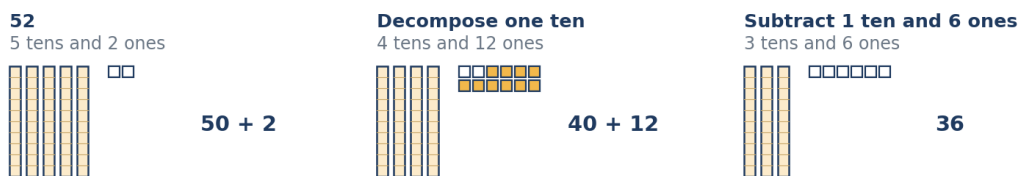
Equality as a relationship. A student says that $8 = 3 + ?$ is backward. The student may be interpreting the equal sign as a signal to calculate rather than as a statement that two quantities are equivalent. Ask what the equal sign means. A bar model shows 8 as the total, made up of 3 and an unknown quantity (Figure 2). Then discuss equations such as $8 = 5 + 3$, $8 = 8$, and $2 + 6 = 5 + 3$ as statements that both sides represent the same quantity.



$8 = 3 + ?$ The total 8 is composed of 3 and an unknown quantity.

Figure 2. The total 8 is composed of 3 and an unknown quantity.

Place value and subtraction. A student calculates $52 - 16 = 44$ by subtracting the smaller digit from the larger in each column: $6 - 2 = 4$ and $5 - 1 = 4$. The difference is 36. The student may be treating each column as a separate single-digit subtraction. Ask what the 5 and the 2 represent. Represent 52 as 5 tens and 2 ones, and decompose one ten into ten ones to get 4 tens and 12 ones. Subtracting 1 ten and 6 ones leaves 3 tens and 6 ones, or 36 (Figure 3). Connect the decomposition of one ten into ten ones to each step of the written procedure, including what is recorded when regrouping.



$52 - 16$: subtract 1 ten from 4 tens and 6 ones from 12 ones. 3 tens and 6 ones is 36.

Figure 3. Decomposing one ten into ten ones: $52 - 16 = 36$.

Key Idea • Identify the idea the student is using, where it works, and where it no longer applies.

Teaching When an Idea Needs to Change

The following considerations can guide instruction when a student is applying an idea that no longer fits. They are not a rigid protocol, and not every error requires all four.

- 1. Identify the idea.** Ask the student to explain the strategy or representation.
- 2. Examine where the idea works.** Identify the quantities and relationships for which the reasoning is valid.
- 3. Examine where it no longer works.** Use a carefully selected task or representation to make the mathematical difference visible.
- 4. Develop the relationship that applies.** Provide explicit instruction, connect models to symbols, and use feedback and varied practice to develop accuracy, efficiency, and flexibility.

Students may develop the idea that division always produces a smaller number because many early division problems involve dividing a positive quantity by a whole number greater than 1, for example, $12 \div 4 = 3$. However, $12 \div \frac{1}{2}$ asks how many units of one-half are contained in 12. Each unit of 1 contains two halves, so 12 units of 1 contain 24 halves. Therefore, $12 \div \frac{1}{2} = 24$. A bar model can show the 12 units of 1, each partitioned into halves.

The goal is not to create confusion; it is to help students understand the relevant mathematical relationship. Hiebert and Carpenter (1992) describe understanding as the connections students make among mathematical ideas, facts, and procedures. Conceptual understanding and procedural fluency develop together, each supporting the other (National Research Council, 2001).

Key Idea • Examine where the student's reasoning applies, where it does not, and the mathematical relationship needed in the new situation.

Using Errors During Instruction

Not every error needs an extended discussion. When a student makes a calculation or recording error and can explain the correct reasoning, specific feedback and further practice are often enough. When the reasoning is unclear, or the same error appears across several problems, ask the student to explain or represent the quantities before deciding what to teach.

An incorrect response can also provide a useful example for discussion, particularly when several students share the same idea. A claim that the order does not matter in division can be examined with $12 \div 4 = 3$ and $4 \div 12 = \frac{1}{3}$: sharing 12 cups of juice among 4 friends gives each friend 3 cups, while sharing 4 cups among 12 friends gives each friend $\frac{1}{3}$ of a cup. In algebra, asking students to explain why incorrect worked examples are incorrect, alongside correct examples, has supported conceptual understanding (Booth et al., 2013). When an error is discussed publicly, keep the focus on the mathematical idea rather than on the student.

Summary

Mistakes and misconceptions provide different kinds of information about students' mathematical knowledge. Teachers can investigate an incorrect response to determine whether students need feedback on a calculation,

additional practice, or instruction that develops a mathematical relationship. When a familiar idea no longer applies, instruction should help students understand why and connect the new relationship to representations, procedures, and practice.

References

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Social Media

Why would a student say that $\frac{7}{8}$ is greater than $\frac{1}{4}$?

The student may be using an idea that works for whole numbers but no longer applies to fractions. An incorrect answer may reflect a calculation mistake or an idea that needs to change. This DMT Insight examines:

- How to distinguish mistakes from misconceptions
- Three examples: fractions, the equal sign, and subtraction
- How to show where an idea works and where it no longer applies
- When to correct directly and when to discuss an error

Read the full Insight at mathsuccess.io/dmtinsights.

Which idea from earlier grades do your students most often apply where it no longer works?

