

DMT INSIGHTS

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HOW MATHEMATICAL UNDERSTANDING DEVELOPS AND LASTS

What research tells us about memory, mathematical relationships, and practice

Introduction

Students develop mathematical understanding by connecting new ideas to prior knowledge, examining relationships among quantities and representations, and practicing what they learn. As students organize and retain these relationships, they become better able to retrieve and apply mathematical knowledge accurately, efficiently, and flexibly.

Cognitive psychology research offers important insights into how this learning occurs. Bruner, Ausubel, and Piaget examined how learners represent, connect, and reorganize knowledge. Sweller's work on cognitive load explains working memory limitations, while research on retrieval practice, spacing, and interleaving helps explain how practice supports retention and application.

This DMT Insight examines how these findings can inform mathematics instruction that develops conceptual understanding and procedural fluency together.

How Do Students Develop and Organize Mathematical Knowledge?

Students interpret new mathematical ideas through what they already understand (Ausubel, 1968). Piaget (1965) described two ways learners adapt their knowledge. Assimilation involves interpreting new information through existing knowledge. Accommodation involves modifying that knowledge when familiar ideas do not adequately explain a new situation. Connecting new ideas to prior knowledge can require substantial thinking and instruction, even when the connection seems familiar.

For example, a student who writes $24 \times 3 = 612$ may have multiplied each digit by 3 and recorded the products side by side. The student may know the multiplication facts but may not be attending to the value of the digits in 24. The answer alone does not establish the student's reasoning; asking the student to explain the calculation provides that evidence.

Representing 24 as 2 tens and 4 ones lets students examine three groups of each. The distributive property connects these relationships: $24 \times 3 = (20 \times 3) + (4 \times 3) = 60 + 12 = 72$ (Figure 1). This example illustrates how prior knowledge

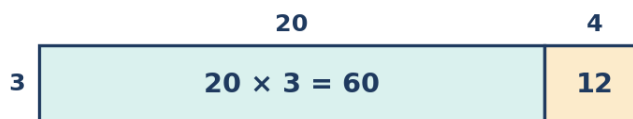
can support a new calculation while also requiring students to reconsider how that knowledge applies to multidigit numbers.

A response

$$24 \times 3 = 612$$

$2 \times 3 = 6$ and $4 \times 3 = 12$,
written side by side

The area model



$$60 + 12 = 72$$

The 2 in 24 is 2 tens, so 2×3 is 6 tens, or 60.

Figure 1. Decomposing 24 into 20 and 4 shows the partial products, 60 and 12.

Bruner (1966) described enactive, iconic, and symbolic ways of representing ideas. In mathematics, students may use physical materials, diagrams, language, and symbols to examine the same mathematical relationships. Teachers can use these representations together and select them based on the mathematics being developed and students' needs. Ausubel (1968) emphasized meaningful learning, in which learners connect new information to relevant ideas they already have, rather than learning in isolation.

Key Idea • Prior knowledge influences how students interpret new mathematics. Student explanations show the relationships they understand and those that need further development.

How Does Mathematical Knowledge Become More Accessible and Flexible?

Working memory has limited capacity to process unfamiliar information (Baddeley & Hitch, 1974; Cowan, 2001). When students learn new mathematics, clear explanations, carefully selected examples, and appropriate representations can help them focus on the relationships that matter. As students develop and organize their knowledge, they can retrieve familiar facts, procedures, and relationships with less demand on working memory.

Sweller (1988) found that problem-solving methods that require learners to search for a solution can place heavy demands on working memory and interfere with learning the structure of a problem. Representations can help students attend to that structure, but they can also add information students must process; whether a physical model or diagram reduces or increases cognitive load depends on its design and how it is used. Instruction and practice help students organize mathematical knowledge so they can retrieve and use familiar facts, procedures, and relationships more efficiently.

What Does Research Tell Us About Practice and Retention?

Understanding a mathematical relationship does not guarantee that students will retain or use it accurately over time. Students also need opportunities to retrieve previously learned knowledge, practice procedures, and determine when particular strategies are appropriate. Spaced and interleaved practice can support retention and strategy selection, while feedback helps students refine their reasoning and procedures.

Retrieval practice. Roediger and Karpicke (2006) found that college students who practiced recalling information from prose passages demonstrated better delayed retention than students who restudied the material. Although this study did not examine mathematics specifically, it provides evidence for the importance of retrieval in learning.

Spaced practice. Cepeda et al. (2006) reviewed research showing that distributing practice over time generally improves long-term retention compared with concentrating practice into one session.

Interleaved practice. Rohrer et al. (2015) found that seventh-grade students who practiced different types of mathematics problems in a mixed sequence performed better on a delayed assessment than students who practiced problems grouped by type. Interleaving can require students to determine which strategy or procedure applies.

Students need opportunities to explain mathematical relationships and to practice using them accurately and efficiently. Understanding and fluency develop together.

Key Idea • Retrieval, spacing, and interleaving can strengthen retention and help students determine when and how to apply previously learned mathematics.

What Are the Instructional Challenges?

Teachers must consider what students already understand, how much unfamiliar information a task requires, and whether students can retrieve and apply what they have learned. An explanation that is appropriate for a student encountering a concept for the first time may provide unnecessary support for a student who has already developed the relevant knowledge. Instruction therefore requires attention to student thinking, feedback, and opportunities for practice over time.

Spaced and interleaved practice can also feel harder to students than blocked practice, even when it produces better retention. Teachers need to decide when students have had enough initial instruction to benefit from mixed practice and how often to revisit important relationships. These decisions depend on the mathematics and on evidence from student work.

What Are the Implications for Curriculum and Teaching Practice?

Instruction should use representations that help students examine the mathematical relationships they are developing. Physical models, diagrams, language, and symbols can each serve this purpose. Explicit instruction, discussion, feedback, and practice help students connect these representations and use the mathematics with increasing accuracy, efficiency, and flexibility.

Formative assessment should identify what students understand about the mathematical structure, such as whether a student is attending to the value of each digit in 24×3 , rather than trying to determine whether a student is assimilating or accommodating. Curriculum and pacing should allow important ideas to be revisited through retrieval and mixed practice across weeks and units.

Conclusion

Cognitive psychology research helps explain how students develop, organize, and retain mathematical knowledge. Prior knowledge shapes how students interpret new ideas, working memory affects how they process unfamiliar information, and distributed practice can support retention.

These findings have important implications for mathematics instruction. Students need opportunities to examine mathematical relationships, connect representations, receive explicit instruction and feedback, and revisit important ideas. Conceptual understanding and procedural fluency develop together as students learn to use mathematics accurately, efficiently, and flexibly.

References

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Social Media

A student can know every multiplication fact and still write $24 \times 3 = 612$.

Students interpret new mathematics through what they already know, and practice affects what they retain and can apply later. Inside this DMT Insight, we look at:

- How prior knowledge supports and sometimes interferes with new learning
- What working memory and cognitive load mean for instruction
- What research shows about retrieval, spacing, and interleaving
- How decomposing 24 into 20 and 4 connects facts, place value, and the distributive property

Read the full Insight at mathsuccess.io/dmtinsights.

Which mathematical relationships do your students still use accurately a month after they learn them?

