

DMT INSIGHTS

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ASSESSING MATHEMATICAL UNDERSTANDING: FROM STUDENT RESPONSES TO INSTRUCTION

Using the DMT assessment framework to examine student work and plan instruction

Introduction

Assessment provides teachers with evidence of what students know and can do mathematically. Examining students' strategies, representations, and explanations helps teachers identify the quantities and relationships students use and decide what instruction they need. The DMT assessment framework helps teachers design tasks that examine different aspects of mathematical knowledge and use student responses to plan instruction.

In *Assessing Student Understanding: A Framework for Testing and Teaching* (Brendefur et al., 2016), we described a framework for examining students' procedural skills, problem solving, conceptual understanding, and reasoning.

DMT Insight 44 examines cognitive demand, while Insight 22 examines state mathematics assessments. Here, the focus is on what student responses reveal about mathematical knowledge and how teachers use that evidence to plan instruction.

Key Idea • The value of assessment lies in the evidence teachers gather about students' mathematical knowledge and how they use that evidence to plan instruction.

The DMT Assessment Framework

The DMT assessment framework describes four task categories organized into three levels. Each category examines a different aspect of mathematical knowledge. The framework draws on earlier work on assessment and mathematical knowledge, including Webb's (2002) depth-of-knowledge levels, de Lange's (1999) framework for classroom assessment, and research on conceptual and procedural knowledge (Hiebert, 1986). It serves a different purpose from Webb's framework. The DMT categories describe the aspect of mathematical knowledge a task examines; Webb's levels describe the cognitive demand a task requires. A DMT category should not be equated with a DOK level.

- **Level 1: Skills and Procedures.** Tasks examine accuracy, efficiency, and procedural fluency. Procedural fluency is an important part of mathematical knowledge and develops alongside conceptual understanding.
- **Level 2: Problem Solving.** Tasks examine how students interpret the quantities and relationships in a problem and select and use a strategy. A problem may allow several valid strategies and still have one correct answer.

- **Level 2: Conceptual Understanding.** Tasks examine how students represent, compose, decompose, and explain mathematical units and relationships.
- **Level 3: Reasoning and Justification.** Tasks examine how students justify mathematical claims, examine generalizations, and evaluate reasoning.

The category depends on the mathematical knowledge and reasoning the task is designed to assess. A model, context, or request for an explanation does not, by itself, place a task in a particular category or raise its cognitive demand. A task's demands can also change as it is carried out in the classroom (Stein & Smith, 1998).

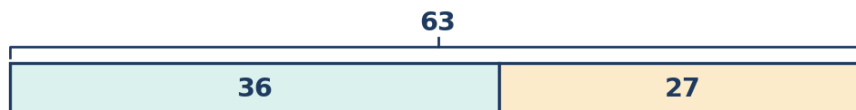
One Topic, Four Tasks: 36 + 27

The matrix below uses one mathematical topic, place value and two-digit addition, so the evidence each task can provide is easier to compare. The last column describes what a teacher can examine in the responses.

DMT category	Assessment task	What can the teacher learn?
Level 1: Skills and Procedures	Calculate $36 + 27$.	Whether the student calculates accurately and efficiently, and which procedure the student uses
Level 2: Problem Solving	A class collected 63 cans over two days. They collected 36 cans on Monday. How many cans did they collect on Tuesday? Show how you determined the missing quantity and explain how the quantities are related.	How the student interprets the known total and part, selects a strategy, and explains how the quantities are related
Level 2: Conceptual Understanding	Use a bar model or place-value representation to show how 36 and 27 compose 63. Explain how ten ones can be composed into one ten.	Whether the student represents tens and ones, composes ten ones into one ten, and explains why the total is 63
Level 3: Reasoning and Justification	A student says, "I added 30 and 20 to get 50, then 6 and 7 to get 13, so the total is 63." Explain why the strategy works. Would the same way of composing tens and ones work for other two-digit numbers, whether or not the ones compose another ten? Justify your answer.	Whether the student can explain why decomposing into tens and ones keeps the total the same, and whether the student can justify a general claim

The Level 2 Conceptual task asks students to show how the quantities compose 63 (Figure 1). The bar model represents 36 and 27 as parts of the total. Decomposing 36 and 27 into tens and ones gives 5 tens and 13 ones. Composing ten of those ones into one ten gives 6 tens and 3 ones. The quantity remains the same.

Bar model: 36 and 27 compose 63



36 + 27 in tens and ones: 5 tens and 13 ones



Compose ten ones into one ten: 6 tens and 3 ones



The bar lengths are proportional. The darker ten is composed from ten ones.

Figure 1. 36 and 27 compose 63; ten ones compose one ten.

Each response provides some evidence, not a complete picture. A student who calculates 63 correctly may or may not be able to explain why composing ten ones into one ten works. A student who represents the quantities accurately may still need practice to calculate efficiently. A task's exact classification also depends on its design and the reasoning it requires.

Classroom Examples from the 2016 Article

The 2016 article described first- and fourth-grade classrooms in which teachers used the framework during instruction.

First grade. Kelli Rich used the framework during a unit on place value, asking students to represent numbers in multiple ways using base-ten pieces. Tasks like these let a teacher see whether students treat ten as a unit, compose and decompose tens and ones, and recognize that different representations, such as 4 tens and 3 ones and 3 tens and 13 ones, show the same quantity.

Fourth grade. Sarah Appleton's students analyzed equivalent fractions using number lines and area models. Tasks like these let a teacher see whether students identify the unit of 1, partition it into equal parts, iterate a fractional unit, and explain why two fractions represent the same quantity. Using a number line or an area model does not by itself make a task Level 3. The category depends on whether students justify a claim or evaluate reasoning.

Using Student Responses to Plan Instruction

Student responses provide evidence of the mathematics students use. A correct answer may come from an efficient procedure, a representation of the quantities, or a strategy the student cannot yet explain. An incorrect answer may involve a calculation error, confusion about the unit, or a relationship that needs further development. Teachers need to examine the work, ask questions when the reasoning is unclear, and use that evidence to plan instruction, feedback, and practice. Eliciting and using evidence of student thinking is an effective mathematics teaching practice described by the National Council of Teachers of Mathematics (NCTM, 2014).

For $36 + 27$, a response of 513 may come from adding the ones and the tens separately and recording both sums without composing a ten. Before deciding what instruction is needed, the teacher can ask the student to explain the written record or represent the quantities using tens and ones. The response may indicate a need to compose ten ones into one ten, connect a representation to the written procedure, or record the composed ten accurately.

Key Idea • Treat a student response as evidence to investigate, not as a diagnosis.

A Process for Teachers and Teams

Teachers and teams can use the framework in four steps: (1) select a mathematical relationship students are learning; (2) design tasks that examine different aspects of that knowledge; (3) examine student strategies, representations, and explanations; and (4) use the evidence to plan instruction, feedback, and practice.

Conclusion

The DMT assessment framework helps teachers examine students' procedural skills, problem solving, conceptual understanding, and reasoning. Student responses provide evidence of the quantities, units, and mathematical relationships students use. Teachers can use this evidence to plan instruction, provide feedback, and develop conceptual understanding and procedural fluency through practice.

References

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Social Media

What does a student's response to $36 + 27$ tell us?

A student may calculate 63 using a written procedure, compose a ten, count on, or represent the quantities with a model. Each response provides different evidence about the mathematics the student is using. This DMT Insight examines:

- How to design tasks that examine different aspects of mathematical knowledge
- How to investigate student strategies and errors
- How student responses connect to units, quantities, and relationships
- How to plan instruction, feedback, and practice from the evidence

Read the full Insight at mathsuccess.io/dmtinsights.

What do your students' strategies tell you about what to teach next?

