

DMT INSIGHTS

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RETHINKING POWERS OF TEN: UNDERSTANDING THE $\times 10$ STRUCTURE

Introduction

Powers of ten are not an isolated topic or a rule for moving digits and adding zeros. They describe the structure of our base-ten number system. Each unit is composed of ten of the unit to its right, and each can be partitioned into ten of the next smaller unit. Understanding this relationship helps students reason about place value, magnitude, decimals, measurement, and scientific notation.

The $\times 10$ Structure of Base-Ten Units

Start with the units, not the exponents. Ten ones compose one ten, ten tens compose one hundred, and ten hundreds compose one thousand. The same relationship runs in the other direction: one one is ten tenths, and one tenth is ten hundredths. Each unit is ten times the unit to its right and one tenth the unit to its left.

Only after students understand this structure does the notation help, writing 10^1 , 10^2 , and 10^3 compactly records a multiplicative structure students have already built with units and quantities. The notation comes after the structure, not before it.

Key Idea • Each base-ten unit is ten times the unit to its right and one tenth the unit to its left. The exponent notation records this structure; it does not replace it.

From Pattern to Structure

Students often notice the pattern before they understand why it works. Seeing that 34, 340, and 3,400 are related is useful. The mathematical goal is to understand that each quantity is ten times the previous one.

The productive question is not what the digits look like, but what changed: the digits, the units, or the quantity? In 34, 340, and 3,400, the digits 3 and 4 stay the same. What changes is the unit each digit counts, and so the size of the

quantity.

Key Idea • Noticing the pattern is a start; the goal is understanding that each quantity is ten times the previous one. Ask what changed: the digits, the units, or the quantity.

Common Misconceptions

Powers-of-ten errors are diagnostic. Three are especially common (Ashlock, 2010).

Multiplying by ten means add a zero. This appears to work with whole numbers, but it does not express the mathematics and breaks down as a general explanation. With decimals, it fails outright, since 3.4 times ten is 34, not 3.40.

The digits move. It is more accurate to say the quantity becomes ten times as large, and each digit now counts a unit ten times as large. In 34, the 3 counts tens; in 340, the 3 counts hundreds; its contribution changes from 30 to 300.

Multiplying by ten means adding ten. A student who thinks 34 becomes 340 by adding ten has not coordinated the multiplicative relationship between adjacent units. The step from 34 to 340 is ten times as large, not ten more.

Key Idea • Multiplying by ten does not add a zero or move digits. The quantity becomes ten times as large, and each digit counts a unit ten times as large.

Effective Teaching Approaches

Four moves keep the focus on units, quantity, and the $\times 10$ relationship rather than on rules about digits.

Build and rename units. Use nested base-ten units: ten ones compose one ten, ten tens compose one hundred, ten hundreds compose one thousand. Ask students to compose larger units and to decompose or partition them into smaller units, and to rename a quantity, such as 340 as 34 tens and as 340 ones (Carbonneau, Marley, & Selig, 2013).

Compare related quantities. Put 34, 340, and 3,400 side by side and ask: How many times as large? What unit does the 3 count in each number? What is the value of the 3? What changed, and what stayed the same?

Connect representations. Use base-ten units, place-value charts, number lines, measurement, and symbols, but keep the purpose clear: each representation should reveal the same $\times 10$ relationship, not simply add another model.

Extend the structure across the unit of one. Continue the pattern past one: 100, 10, 1, 0.1, 0.01. The unit of one anchors the system; students iterate units to the left and partition units to the right. This bridges whole-number place value and decimals.

Key Idea • Build and rename units, compare related quantities, connect representations to one relationship, and extend the structure across the unit of one.

Conclusion

Understanding powers of ten means understanding how units are related. Each unit is ten times the unit to its right and one tenth the unit to its left. Students need opportunities to compose, decompose, compare, and rename these units before powers-of-ten rules become shortcuts (Brez & Allen, 2016; Saenz et al., 2023).

The goal is not for students to remember that digits move or that zeros are added. The goal is for them to explain why the quantity changes and how that change is represented in the base-ten system.

When students understand the $\times 10$ structure, place value, decimals, measurement, and scientific notation become connected ideas rather than separate procedures.

References

Ashlock, R. B. (2010). Error patterns in computation: Using error analysis to help each student learn. Pearson.

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Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of the efficacy of teaching mathematics with concrete manipulatives. *Journal of Educational Psychology*, 105(2), 380–400.

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Social Media

Ask a student what happens when 34 is multiplied by ten.

Many will say, add a zero. They may get 340, but the explanation tells us little about whether they understand what happened to the quantity. The quantity is now ten times as large: the 3 that counted tens now counts hundreds, and the 4 that counted ones now counts tens.

Inside this overview, you will learn:

- Why powers of ten describe the $\times 10$ structure of base-ten units, not a rule for moving digits
- How to separate the digit, the unit, and the quantity when a number is multiplied by ten
- Why add a zero and the digits move: break down as explanations
- How the unit of one connects whole numbers to decimals: 100, 10, 1, 0.1, 0.01

Read the full overview at mathsuccess.io/dmtinsights.

When your students multiply by ten this week, can they explain what happened to the quantity, not just the digits?

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