

DMT INSIGHTS

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WHY MEASUREMENT MATTERS: HOW THE UNIT GIVES A NUMBER ITS MEANING

Introduction

A second grader measures a book with paper clips and reports 9. She measures it again with craft sticks and reports 3. When asked which measurement is correct, she chooses 9 because it is more. Both counts are correct. What she has not yet coordinated is the relationship between the size of the unit and the number of units; without that relationship, neither number tells her how long the book is.

Measurement is foundational to later mathematical learning because numbers describe quantities. To measure, a student identifies an attribute, chooses a unit of that attribute, iterates the unit with no gaps or overlaps, and reports how many units compose the quantity (Lehrer, 2003). This same structure appears throughout mathematics. The number line is a measurement model, fractions partition a unit of 1 into equal-sized units, and place value composes units of different sizes. Measurement gives a number meaning: the number counts units of something. A number line built from iterated units supports the representation of magnitude that whole numbers and fractions share (Siegler, Thompson, & Schneider, 2011).

Measurement is also demanding. Students can read a ruler, name standard units, and report correct measurements while still treating the ruler as a row of numerals rather than as equal units counted from 0 (Kamii & Clark, 1997; Solomon, Vasilyeva, Huttenlocher, & Levine, 2015). This overview examines how measurement begins with an attribute rather than a number, how the unit gives the number meaning, how the ruler and the number line record iterated units, and how measurement extends to fractions, decimals, and place value.

Key Idea • A measurement is a count of units. A correct number shows that a student counted. It does not show whether the student knows what was counted or how large each unit was.

Attribute Before Number

Measurement begins before any number appears. A student first decides what to measure: the length of a table, the amount a container holds, or the weight of a stone. Young children compare these attributes directly by placing two objects side by side from a common starting point, and indirectly by using a third object, such as a string, to compare two objects that cannot be moved together (Lehrer, 2003). Indirect comparison asks the student to reason that if the

string is longer than the table and the table is longer than the desk, the string is longer than the desk. That inference, called transitivity, later allows a ruler to stand in for the objects it compares.

Comparison gives measurement its purpose. A student who has compared two lengths directly knows what longer means before a number states how much longer. In the curriculum V. V. Davydov developed, first graders compare quantities such as lengths, areas, and volumes and record the relationships among them before working with numbers. Number then enters as a relationship between two quantities: the quantity being measured and the unit (Schmittau & Morris, 2004).

Comparison deserves time in instruction because students first need to understand which attribute they are measuring. A student who has not separated length from area, or the height of an object from the distance around it, counts units of the wrong attribute and reports a correct-looking number for a question no one asked.

Key Idea • Before a number can describe a quantity, the student has to know which attribute is being compared. Direct and indirect comparison establish the attribute and the logic the measuring tool depends on.

The Unit Gives the Number Its Meaning

A measurement is a count of units, and the count has no meaning without the unit. Three ideas are central to understanding the unit (Lehrer, 2003).

Equal-sized units. Every unit in a measurement is the same size, or the count does not describe the quantity.

Iteration. The unit is repeated end to end from a starting point, with no gaps or overlaps.

Unit size and unit count. For the same quantity, a larger unit produces a smaller count, and a smaller unit produces a larger count.

The student in the opening example has not yet coordinated unit size with unit count. Carpenter and Lewis (1976) found that young children who could state the inverse relationship between unit size and the number of units did not apply it when they compared two objects measured with different units. They compared the counts instead. Knowing the relationship and using it to reason are different. Students need tasks that require them to use it.

The usual instructional sequence moves from direct comparison to nonstandard units, then to standard units, and finally to rulers. Clements (1999) questioned that order. Nonstandard units can make the need for equal-sized units visible. However, when students measure with hands, feet, and paper clips of different sizes, they may take away that measurement gives different answers. Standard units and rulers can appear early, alongside nonstandard units, when instruction keeps attention on the unit being iterated. Unit iteration also does not have to wait for a developmental stage. Hiebert (1981) found that first graders could learn to iterate a unit correctly before they showed conservation of length on Piagetian tasks. However, other measurement concepts did depend on conservation and transitivity.

Standard units then have a reason to exist. When two students measure the same table with different units and report different numbers, the class has a problem that only a shared unit resolves.

Key Idea • The count has no meaning without the unit. A larger unit gives a smaller count for the same quantity, and students need to reason with that relationship, not only state it.

CLASSROOM SNAPSHOT

Teacher: “Ana measured the book and got 9. Leo measured the same book and got 3. Whose measurement is correct?”

Ana: “Mine. Nine is more than three.”

Leo: “It is the same book. I used the sticks.”

Teacher: “So how can both of you be correct?”

Leo: “The sticks are longer, so you need fewer of them.”

Ana: “So if I used something smaller than a paper clip, I would get more than 9?”

Reading the Ruler as Iterated Units

A ruler records iterated units. Each interval between two marks is one unit, the numeral at a mark counts the units from 0 to that mark, and the length of an object is the number of units between its two ends. Students often read a ruler as a row of numerals rather than as equal units. Kindergarten and second-grade children measured accurately when they aligned an object with the start of a ruler, and they performed at chance when they shifted the object away from 0, even though they succeeded on shifted problems when they could count separate units (Solomon et al., 2015).

Barrett and colleagues followed second- and third-graders through a longitudinal teaching experiment. They found that counting intervals as units was not enough on its own. To measure consistently, students had to coordinate the intervals, the hash marks, and the number labels on the ruler at the same time (Barrett et al., 2012). The ruler records iteration in a compact form, and students have to see the iterations in it to use it with understanding.

Used with understanding, the ruler supports reasoning. Nunes, Light, and Mason (1993) found that children reasoned about length more successfully with a ruler than with a piece of string, and that developmental level alone could not explain the difference. The instructional task is to make the ruler units visible to students.

The number line extends the ruler. Each number on it names a location and its distance from 0, measured in units of 1. Students who read the number line as a measurement model can locate whole numbers, fractions, and decimals by the same reasoning. Siegler, Thompson, and Schneider (2011) place that shared account of magnitude at the center of numerical development.

Key Idea • A ruler records units iterated from 0. A student who counts the intervals between the ends of an object is using the ruler as a measurement tool, and the same reasoning builds the number line.

Measurement Carries Into Fractions, Decimals, and Place Value

When a unit is too large to measure a quantity exactly, students partition it. Partitioning the unit of 1 into four equal-sized units creates units of one-fourth, and three-fourths is a count of three of those units. The denominator names

the unit size, and the numerator counts how many of those units there are. This is the same unit-and-count structure students use to measure length, now applied to units smaller than 1.

This measurement structure also supports students' understanding of fraction magnitude. Second- and third-graders randomly assigned to number line training improved on an untrained fraction magnitude comparison task. In contrast, students who received area model training improved only on area model tasks (Hamdan & Gunderson, 2017). The number line locates a fraction as a distance from 0 in units of a given size, and that measurement reading carried over to the new task.

Place value uses this same structure with units of different sizes. Ten units of 1 compose one unit of 10, and one unit of 1 partitioned into ten equal-sized units creates units of one-tenth. The metric system follows the same relationship: 10 millimeters compose 1 centimeter, and 100 centimeters compose 1 meter. Students who reason with units of different sizes in measurement are already reasoning with the structure that multidigit numbers and decimals require.

Measurement provides a direct path from action to representation to symbol. Students iterate a physical unit, represent those iterations on a number line or bar model, and then record the measurement. Bruner (1966) described this progression from enactive to iconic to symbolic representation. In measurement, the symbol records an action students have already performed.

Key Idea • Fractions, decimals, and place value measure with units of different sizes. The unit-and-count structure students build in measurement is the structure they use there.

Conclusion

Measurement makes the meaning of number especially visible. A number that results from measuring counts units of an attribute, and the size of those units determines what the count means. The same structure supports students' understanding of number lines, fractions, decimals, and place value. Without it, each of these can appear to students as a new and unrelated set of procedures.

Instruction that builds this structure is specific. It establishes the attribute through direct and indirect comparison before it assigns a number. It asks students to iterate equal-sized units with no gaps or overlaps and to reason about how the count changes when the unit changes. It teaches the ruler as a record of units iterated from 0 and extends that reading to the number line. It asks the same two questions whenever a student reports a measurement: What is the unit, and how many of those units are there?

Key Idea • A measurement names a unit and how many of those units compose a quantity. Students who hold that structure carry it into the number line, fractions, decimals, and place value.

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Social Media

Which measurement is correct, 9 or 3?

A second grader measures a book with paper clips and gets 9. A classmate measures the same book with craft sticks and gets 3. When asked whose measurement is correct, the first student says hers because 9 is more than 3. Both counts are correct. What is missing is the relationship between the unit size and the number of units.

A measurement is a count of units, and the count has meaning only when the unit is named. The same structure returns when students read a ruler, locate a number on a number line, name a fraction, and compose units in place value. This Insight examines how measurement lays the foundation for that structure.

- Why students compare attributes before they assign numbers
- How the size of the unit changes the count, and why students need to reason with that relationship
- Why a ruler is a record of units iterated from 0
- How fractions, decimals, and place value extend the same unit-and-count structure

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When your students measure this week, can they tell you what the unit is and how many of those units compose the length?



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