

DMT INSIGHTS

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MATH FACT FLUENCY: BUILDING FACTS FROM RELATIONSHIPS

From derived strategies to efficient recall

Introduction

Math fact fluency is more than answering quickly. A fluent student can recall a fact or derive it efficiently from a known relationship. Over time, many of those relationships become increasingly automatic.

Consider $8 + 7$. A student might recall 15 immediately. Another might reason from $8 + 8 = 16$ and subtract 1. Both responses can show fluency when the derivation is accurate and efficient. More importantly, the second response reveals a mathematical relationship that students can reuse with other numbers.

The instructional goal is not to choose between understanding and automaticity. Students need both. Mathematical relationships give students something to reason from, while continued practice makes those relationships increasingly efficient to access.

Key Idea • Fluency is not speed alone. It includes accurate and increasingly efficient recall or derivation of facts from known mathematical relationships.

What Fluency Includes

People often treat the words used to describe fluency as interchangeable. Four of them name different things:

- **Accuracy.** The answer is correct.
- **Efficiency.** The student can reach the answer without unnecessary steps.
- **Flexibility.** The student can use and adapt known relationships.
- **Automaticity.** Access to a fact or relationship requires little conscious effort.

Fluency develops as accuracy, efficiency, flexibility, and automaticity increasingly work together. Students should not have to reason through every basic fact forever. The goal is increasingly efficient and automatic access. However, automaticity can develop from relationships students understand rather than from memorizing every fact as an isolated item.

As facts and useful relationships become more automatic, students can devote more attention to the other mathematics in a problem. This is one reason fluency matters: basic calculations should not consume all of the student's available attention during more complex work (Geary, 2011; Sweller et al., 2011).

Key Idea • Fluency develops as accuracy, efficiency, flexibility, and automaticity increasingly work together.

When Fact Practice Becomes Disconnected From the Mathematics

Fact practice can become disconnected from the mathematics in three ways.

- **Isolated facts.** Students may remember individual answers without seeing relationships among them. Kilpatrick et al. (2001) describe conceptual understanding and procedural fluency as interwoven strands and note that knowledge learned as isolated pieces is more easily forgotten.
- **Speed without strategy.** A correct, quick response tells us something about retrieval but not necessarily what mathematical relationships the student can use when a fact is forgotten. Teaching that attends explicitly to connections among facts, procedures, and ideas supports conceptual understanding (Hiebert & Grouws, 2007).
- **Strategies without movement toward fluency.** The opposite problem also occurs. Students should not have to reconstruct every basic fact indefinitely. Efficient strategies should become increasingly compressed and automatic with practice.

The useful question is not whether students should memorize facts or understand them. Students need efficient access to basic facts and mathematical relationships that let them derive, connect, and reconstruct those facts. Understanding and fluency develop together. Research on conceptual and procedural knowledge describes the relation as bidirectional: each supports growth in the other (Rittle-Johnson et al., 2015).

Key Idea • Students need efficient access to basic facts and relationships they can use to derive, connect, and reconstruct them. Understanding and fluency develop together.

Build Unknown Facts From Known Facts

Children derive unknown facts from facts they already know. Research on addition and subtraction has long described derived facts as a strategy children use between counting and recall (Carpenter & Moser, 1983). Baroody (2006) argues that mastery of the basic number combinations grows from number sense, the patterns and relationships among the combinations, rather than from memorizing them as unrelated items.

DMTI teaches derived facts as known relationships used to determine unknown facts (Brendefur & Strother, 2021). Four actions organize the work:

- **Use a known fact.** $8 + 8 = 16$, so $8 + 7$ is one less: 15.
- **Compose or decompose.** $7 + 6 = 7 + 3 + 3 = 10 + 3 = 13$.
- **Use a benchmark.** $9 \times 6 = 10 \times 6 - 6 = 54$.
- **Use a relationship between operations.** If $7 \times 8 = 56$, then $56 \div 8 = 7$.

The same structural language applies across the operations. In multiplication, students can reason about a unit and the number of times it is iterated. If 6×7 is known, then 7×7 can be understood as one additional unit of 7: $6 \times 7 = 42$, so

$7 \times 7 = 42 + 7 = 49$. In addition, students can compose and decompose quantities around useful benchmarks such as 5 and 10. In division, known multiplication relationships provide a structure for deriving division facts.

Key Idea • A derived fact is not a trick to memorize. It is a known relationship used to determine an unknown fact.

Make the Relationship Visible

A model earns its place when it makes a fact relationship visible. A ten-frame can show why $8 + 7$ can be decomposed into $8 + 2 + 5$. An area model can show why 7×8 can be built from 5×8 and 2×8 . A number line can show how a known quantity is composed or adjusted (Brendefur & Strother, 2021).

The goal is not to use more representations. Instead, use a small set of representations repeatedly so students can see the same relationships across facts.

Language can direct attention to the same relationships. Asking students to explain which known fact they used, and how it changed, keeps attention on the relationship rather than on the answer alone (Chapin et al., 2009).

Contexts can help establish the meaning of an operation and the quantities being related (Van de Walle et al., 2013). As those relationships become established, students also need focused opportunities to derive, retrieve, and practice facts efficiently.

Key Idea • Use a small number of models repeatedly so students see the same relationships across facts.

Practice Should Make Relationships More Efficient

Practice matters. Students need repeated opportunities to retrieve facts and use known relationships until those relationships become increasingly efficient and automatic. The National Mathematics Advisory Panel (2008) concluded that proficiency with whole-number operations depends on sufficient practice to develop automatic recall of the basic facts.

However, practice does not have to mean treating every fact as an isolated item. Practice can ask students to retrieve a known fact, derive a related fact, explain the relationship, and then use that relationship repeatedly until retrieval becomes efficient. The goal is movement from reasoning to increasingly efficient reasoning and, for many facts, automatic recall.

The instructional progression is: build the relationship, derive from known facts, practice the relationship, increase efficiency, and develop automaticity. This is not a rigid sequence. Students move back and forth between reasoning and retrieval as new facts and operations are introduced. The National Council of Teachers of Mathematics (2014) describes this work as building procedural fluency from conceptual understanding.

Key Idea • Practice makes a known relationship efficient. For many facts, efficient reasoning becomes automatic recall.

Start From What the Student Already Knows

Students who struggle with fact fluency often need the relationships made more explicit, not simply more repetitions of the same isolated facts. Models can make a relationship visible, teacher questioning can focus attention on that relationship, and carefully selected practice can help make it increasingly efficient.

Intervention should begin by asking what the student already knows. If $6 + 6$ is known, can the student use it to determine $6 + 7$? If 5×8 is known, can the student use it to determine 6×8 ? The known relationship becomes the starting point for the next fact.

A meta-analysis of 42 intervention studies for students with learning disabilities found the largest effects for explicit instruction and for teaching students strategies they can apply across problems. Visual representations and having students verbalize their reasoning also had significant effects (Gersten et al., 2009).

Key Idea • The known relationship becomes the starting point for the next fact.

Conclusion

Fact fluency matters because students need efficient access to basic number relationships as the mathematics becomes more complex. However, efficiency does not have to begin with isolated memorization.

Students can build unknown facts from known facts by composing, decomposing, iterating, using benchmarks, and connecting inverse operations. With continued practice, those relationships become increasingly efficient, and many become automatic.

The goal is not for students to reason slowly through every fact forever. Nor is it to memorize every fact without a relationship behind it. Fluency develops when students have mathematical relationships to reason from and enough practice for those relationships to become efficient.

Key Idea • If a fact is forgotten, understanding gives the student somewhere to rebuild it from.

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Ask two students for $8 + 7$. One says 15 at once. The other says, “ $8 + 8$ is 16, so $8 + 7$ is 15.” Which student is fluent?

Both can be. Fluency is more than speed. It includes accurate and increasingly efficient recall or derivation of facts from relationships students already know. With practice, many of those relationships become automatic. Inside this DMT Insight, we look at:

- What accuracy, efficiency, flexibility, and automaticity each mean
- How students build unknown facts from known facts
- When a model makes a fact relationship visible
- How practice makes a relationship efficient

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