

DMT INSIGHTS

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Building Place Value Understanding

Introduction: Naming Places, Understanding Units

Students can often name the places in a number long before they understand the units those places represent. Place value understanding develops when students recognize that a multi-digit number is one quantity composed of nested base-ten units. Ten units of one compose one unit of ten; ten units of ten compose one unit of a hundred. That structure, not simply naming the places, is what students need for later computation and decimal reasoning (Fuson, 1990; Ross, 1989; Hiebert & Wearne, 1992).

The Nature of Place Value

Place value is a system of nested units. Ten units of one compose one unit of ten, ten units of ten compose one unit of a hundred, and the relationship continues in both directions. A digit tells how many of a particular unit are being counted. Its position identifies the unit; its value depends on both the digit and that unit (Fuson, 1990; Hiebert & Wearne, 1992).

Key Idea • Place value is a system of nested units. A digit tells how many of a unit are being counted; its position names the unit, and its value depends on both.

What Students' Errors Tell Us

Place-value errors are rarely careless. Each one shows whether a student is reading a digit or reasoning about units (Kamii, 1986; Ross, 1989).

A student who says 340 has 4 tens rather than 34 tens is reading the digit rather than counting units. A student who sees 36 as a 3 and a 6 has not yet coordinated the digits with 3 units of ten and 6 units of one. A student who borrows without knowing what changed may follow a correct procedure without seeing that one unit of a hundred has been partitioned into ten units of ten. A student who says multiplying by ten means add a zero may know a pattern without understanding the $\times 10$ relationship among adjacent units.

Key Idea • Place-value errors are diagnostic. Each one shows whether a student is reading digits or reasoning about the units a number is composed of.

Representations

The goal is not to use more representations. The goal is to help students see the same mathematical structure across representations (Bruner, 1966; Van de Walle, 2007).

Base-ten units make unit size and composition visible. Bar and length models make composed quantities visible. Number lines make magnitude and relative position visible. Place-value charts make the relationship among adjacent units visible. Symbols record the structure students have already reasoned about (Fuson, 1990; Hiebert & Wearne, 1992).

Center the place-value chart on the unit of one. From that unit, students can iterate to build tens, hundreds, and thousands, and partition to build tenths and hundredths. This makes the $\times 10$ relationship visible across the entire base-ten system.

Key Idea • The aim is not more representations, but the same structure seen across them. Symbols come last, recording reasoning students have already done.

Grounding in Real-World Contexts

Context gives the units meaning. Measurement, money, distance, and large quantities can give students something to reason about before the notation becomes the focus. The context should clarify the quantity and the unit, not simply decorate the mathematics (Gravemeijer, 1994; Van de Walle, 2007).

Developing Benchmark Understanding

Benchmarks help students reason about magnitude. Ten, 100, 1,000, one-half, and other familiar quantities give students reference points for locating, comparing, and estimating numbers (Hiebert & Wearne, 1992; Van de Walle, 2007).

Language Shapes What Students Attend To

Many students can follow rote procedures for place-value calculations but struggle to explain their reasoning or apply it to new situations (Kamii, 1986; Hiebert & Wearne, 1992). Kamii (1986) found that students often treat multi-digit numbers as single digits placed side by side, which suggests that language focused only on digit positions is inadequate. Research supports asking about complete units, such as how many complete tens are in a number, rather than the value of a digit in a place: a student asked how many tens are in 123 comes to see 12 complete tens and 3 ones (Fuson, 1990; Brendefur et al., 2024).

The shift is small and specific. Instead of asking which digit is in the tens place, ask how many tens are in 340. Instead of saying borrow from the hundreds, say partition one unit of a hundred into ten units of ten. Instead of saying add a zero, ask what happens to the value when the unit becomes ten times as large.

Key Idea • The question shapes the reasoning. Asking how many tens are in 340 reveals how a student thinks about the quantity; asking which digit is in the tens place checks only a label.

Conclusion

Place value is more than knowing where a digit sits. Students need to understand the unit the digit counts, the quantity those units compose, and the multiplicative relationship among adjacent units.

The instructional shift is straightforward: ask students to compose and decompose units, rename quantities, locate numbers by magnitude, and explain how the value changes as the unit changes. Models and language should make that structure visible.

When students understand the structure of the base-ten system, the procedures that follow have something to attach to.

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What does a correct answer about place value actually show?

A student points to the 4 in 340 and correctly names the tens place. Ask how many tens are in 340, and the same student may answer four. Naming the place is not the same as understanding the units the number is built from.

Inside this overview, you will learn:

- Why place value is a system of nested units, not a set of named columns
- What a student's errors reveal about the units they are or are not reasoning about
- How to help students see the same structure across base-ten blocks, number lines, and charts
- Why asking how many tens are in 340 reveals more than which digit is in the tens place

Read the full overview at mathsuccess.io/dmtinsights.

When your students work with a multi-digit number this week, are they naming places or reasoning about units?

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