

DMT INSIGHTS

Issue 10 · 2026 · by Jonathan Brendefur, PhD, and Associates



THE POWER OF DIVISION: UNDERSTANDING WHAT THE QUOTIENT COUNTS

Introduction

A student writes $18 \div 3 = 6$. The answer is correct, but what does the 6 count?

If 18 chickens are placed into coops that hold 3 chickens each, the 6 counts coops. Suppose 18 chickens are shared equally among 3 coops; the 6 counts chickens in each coop. The equation is the same. The mathematical question is not.

That distinction matters. In a U.S. longitudinal sample, knowledge of division at ages 10 to 12 was one of the strongest elementary mathematics predictors of high school mathematics achievement, after the researchers accounted for other mathematical knowledge, general intellectual ability, working memory, and family income and education (Siegler et al., 2012). However, understanding division requires more than calculating a quotient. Students need to understand the relationship among the total, the unit, and the number of iterations.

This Insight examines the two structures of division, the language that makes those structures visible, and how students can move from informal strategies toward increasingly efficient procedures.

Key Idea • Division relates a total, a unit, and an iterator. A correct quotient does not necessarily show whether the student knows what the quotient counts.

Total, Unit, and Iterator

The words total, unit, and iterator make the structure of multiplication and division visible. They do not replace dividend, divisor, and quotient. They describe what the quantities are doing in the relationship (Brendefur et al., 2013; Fosnot & Dolk, 2001). Multiplication composes a total by iterating a unit, so six threes names six iterations of the unit 3, composing a total of 18. Division reverses that relationship by finding either the unit or the number of iterations.

Multiplication: $\text{iterator} \times \text{unit} = \text{total}$

Total (dividend). The entire quantity being divided.

Unit. The size of each equal quantity.

Iterator. The number of iterations of the unit. It tells how many times the unit is counted.

The dividend is always the total. The symbols alone do not determine which quantity is the unit and which is the iterator. The situation does.

Measurement division: $\text{total} \div \text{unit} = \text{iterator}$

Partitive division: $\text{total} \div \text{iterator} = \text{unit}$

Without context, $18 \div 3$ can be read as: How many units of 3 compose 18? Six iterations of the unit 3 compose the total 18.

$$18 \div 3 = 6$$

$$6 \times 3 = 18$$

Division and multiplication describe the same multiplicative relationship from different directions. This language keeps attention on that relationship. Students can ask what the total represents, what quantity is being treated as one unit, and how many times that unit is iterated. The same language works when division extends beyond whole numbers. Familiar classroom language such as equal groups remains useful when it names a representation or a problem type, and students keep their own words while the explanatory language stays precise.

Key Idea • The dividend is always the total. The first question a student answers is which of the unit and the iterator the divisor names, and the situation supplies the answer.

Two Situations, One Relationship

The same division equation can represent two different questions, depending on what is known and what is unknown (Fischbein et al., 1985; Kouba, 1989). A student who distinguishes the two situations can explain what the quotient represents, not simply compute it.

Measurement division, also called quotitive division, asks how many units of a known size compose the total. A farmer has 18 chickens and puts them in coops that hold 3 chickens each. The unit is known. The iterator is unknown, and the quotient counts coops. Students model this situation by iterating the unit along a number line or by subtracting it repeatedly.

$\text{Total} \div \text{unit} = \text{iterator}$

$$18 \text{ chickens} \div 3 \text{ chickens per coop} = 6 \text{ coops}$$

Partitive division, also called fair sharing division, asks how large each unit must be when the total is partitioned into a known number of equal units. The same farmer distributes 18 chickens equally among 3 coops. The iterator is known: 3 tells how many equal units the total is partitioned into. The unit is unknown, and the quotient is the number of chickens in each coop. Students model this situation by partitioning a bar model into 3 equal units or by dealing out one at a time, the strategy children bring to division before instruction (Squire & Bryant, 2002).

$\text{Total} \div \text{iterator} = \text{unit}$

$$18 \text{ chickens} \div 3 \text{ coops} = 6 \text{ chickens per coop}$$

Both situations are written as $18 \div 3 = 6$, and in both the 6 is correct. In one, the 6 counts coops; in the other, it counts chickens. If a student cannot say which, the correct answer tells us less about what the student understands.

Same equation, different question	Measurement	Partitive
Known	Total and unit	Total and iterator
Unknown	Iterator	Unit
$18 \div 3 = 6$	How many units of 3 compose 18?	If 18 is partitioned into 3 equal units, how large is each unit?
The quotient counts	The number of units	The size of each unit

Students do not necessarily reason about these two structures in the same way. Research has found different strategies for partitive and measurement situations, making experience with both important (Fischbein et al., 1985; Kouba, 1989). Measurement problems draw repeated subtraction, and partitive problems draw dealing out (Kouba, 1989; Mulligan & Mitchelmore, 1997). Young children also grasp the relationship among the total, the divisor, and the quotient on tasks that require no calculation, placing conceptual understanding ahead of procedure (Correa et al., 1998).

The two situations are not equally available to students. Fischbein et al. (1985) found that students hold an implicit model of division as sharing, and that the model constrains what they accept as a division problem. The asymmetry reaches teachers as well. Roche and Clarke (2013) asked 378 primary teachers to represent division and to match story problems to a given form, and found them markedly more familiar with partitive than with quotitive division.

Key Idea • Sharing fixes the iterator and finds the unit. Measuring fixes the unit and finds the iterator. One relationship, two situations, and the quotient counts something different in each.

CLASSROOM SNAPSHOT

Teacher: “A farmer has 18 chickens. The coops hold 3 chickens each. How many coops does the farmer need?”

Marisol: “Six. I did 18 divided by 3.”

Teacher: “What does the 6 tell you?”

Marisol: “Six chickens in each coop.”

Teacher: “And if the farmer had 3 coops and shared the 18 chickens evenly?”

Devon: “That is 18 divided by 3, too. However, then the 6 is the chickens, not the coops.”

Teacher: “So what tells you which one the 6 is?”

Devon: “The story. You have to know if they told you how many coops or how many go in a coop.”

From Informal Strategies to Formal Procedures

Students' early strategies contain mathematics that can be formalized rather than replaced. Repeated subtraction, repeated addition, sharing, and counting can develop into chunking, partial quotients, and eventually a standard algorithm. When the algorithm comes before this reasoning has developed, students may learn the steps without understanding the relationships those steps represent (van Putten et al., 2005).

The development is not a rigid sequence of stages. It moves from informal reasoning, to making the relationship visible, to chunking known relationships, to connecting representations, to formalizing, and to developing fluency. This is consistent with what Realistic Mathematics Education describes as progressive mathematization (Treffers, 1987). Contexts generate the models, and models become tools for thinking rather than illustrations of a result (Fosnot & Dolk, 2001).

Build the relationship. Students model the situation directly. They deal out one at a time for sharing, and they iterate or repeatedly subtract the unit for measuring. A number line makes the iterations of the unit visible, and a bar model makes the partitioning of the total visible.

Use known relationships. Students stop iterating one unit at a time and take several at once by chunking, doubling, halving, and using multiples. A ratio table records the chunks as partial quotients and keeps the relationship between the iterator and the total in view. To divide 144 by 8, a student may record that 10 units of 8 compose 80, that 8 more units of 8 compose 64, and that 80 and 64 compose 144, so the iterator is 18. Van Putten et al. (2005) followed Dutch students for five months. They found that initially diverse chunking strategies converged on high-level chunking, in which students choose chunks by place value, and that how far a student progressed depended on prior knowledge and on the instructional materials.

Formalize the reasoning. The standard algorithm connects to the same decomposition and quotient relationships students use in partial quotients. It records those relationships more compactly. The algorithm should record mathematics students can already reason about; it should not introduce an entirely new way of thinking. Students who understand the reasoning behind the algorithm are better positioned to reconstruct it when they forget a step.

Key Idea • The standard algorithm records, more compactly, relationships students already use in chunking and partial quotients. It formalizes reasoning; it does not replace it.

Why the Distinction Matters Later

Students often develop an implicit model of division as sharing. That model works in many early problems and becomes limiting as the numbers and situations change (Fischbein et al., 1985).

Expecting the quotient to be smaller than the total. A student who reads every division as sharing expects the answer to be less than the total, which holds when the divisor is a whole number greater than 1 and fails as soon as it is not. When asked to divide 3 by one-half, such a student may answer one and one-half. The measurement reading asks: How many units of one-half compose 3? Six one-half units compose 3, so $3 \div \frac{1}{2} = 6$.

Rejecting a divisor larger than the dividend. The same implicit model makes $2 \div 5$ look impossible. Reading it as sharing 2 among 5 restores it, and the answer is a unit smaller than 1.

The measurement interpretation makes fraction division reasonable before introducing a reciprocal procedure. “Invert and multiply” records a relationship; it should not be the first explanation of that relationship.

Key Idea • Dividing by one-half asks how many one-half units compose the total. The reciprocal procedure records that relationship. It is not the first explanation.

What to Look For

A correct quotient does not necessarily show how a student is reasoning. These questions make the reasoning visible.

- Can the student identify the total?
- Can the student identify the unit?
- Can the student tell what is being iterated or partitioned?
- Can the student explain what the quotient counts?
- Can the student connect the division equation to multiplication?
- Can the student use known relationships to build the quotient?
- Can the student connect those relationships to the written algorithm?

Two errors show which question a student has not yet answered.

Reporting a remainder without naming what it counts. A student who reports a remainder of 2 without saying what the 2 counts may have stopped reading the situation. What is left over is a quantity of the total, less than one more complete unit, and whether to round up, round down, or report a fraction depends on what the iterator counts.

Treating division and multiplication as separate procedures. A student who does not hold the inverse relation may check a quotient by dividing a second time rather than by multiplying the unit by the iterator. That relation extends directly to fractions, where a fraction is itself a division (Jordan et al., 2013).

Key Idea • Each question asks for the same structure: what the total is, what the unit is, what is being iterated or partitioned, and what the quotient counts.

Conclusion

Division makes relationships among quantities especially visible. The structure that makes $18 \div 3$ readable makes a fraction readable, because three-fourths is 3 divided by 4. It makes a rate readable, because miles per hour measures a total in units of one hour. It also makes proportional reasoning readable, because a ratio holds a unit constant while the iterator varies. Without that underlying structure, fractions, rates, and proportional reasoning can appear to students as new and unrelated procedures.

Instruction that builds the structure is specific. It names the total, the unit, and the iterator. It gives both situations from the beginning rather than teaching sharing first and measuring later. It lets students iterate, partition, and chunk, and it connects that reasoning to the algorithm. It asks what the answer counts every time, not only when the answer is wrong.

Key Idea • Division is one multiplicative relationship with two possible unknowns. Students who hold that relationship extend it to fractions, rates, and proportional reasoning; without it, each of those can appear as a new and unrelated procedure.

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Social Media

$18 \div 3 = 6$. What does the 6 count?

If 18 chickens are placed in coops that hold 3 each, the 6 counts coops. If 18 chickens are shared among 3 coops, the 6 counts chickens. Same equation. Same quotient. Different mathematical question.

Division makes more sense when students can identify the total, the unit, and what is being iterated or partitioned—not just carry out a procedure. Inside this DMT Insight, we look at:

- The two structures of division and what the quotient counts in each
- How a number line and a bar model make each structure visible
- How informal strategies can develop into efficient procedures
- Why the measurement interpretation matters when the divisor is a fraction

Read the full Insight at mathsuccess.io/dmtinsights.

Ask your students tomorrow: What does the quotient count?

