

DMT INSIGHTS

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UNLOCKING MATH SUCCESS: FOUNDATIONS IN NUMBER SENSE

Introduction

Early mathematics education shapes students' long-term mathematical understanding and problem-solving abilities. A key component of this foundation is developing number sense in the early years. This overview examines the critical aspects of number sense, its importance in early mathematics education, and effective strategies for fostering it in young learners. Number sense refers to a well-organized conceptual framework of number information that enables a person to understand numbers and number relationships and to solve mathematical problems flexibly. It is a foundational skill that develops gradually as students work with numbers and gain flexibility in their thinking (Berch, 2005; Dehaene, 1997). Research in mathematics education highlights several critical themes that influence the development of number sense in early learners. A significant focus is on the balance between procedural fluency and conceptual understanding and the instructional strategies educators employ to foster deep mathematical thinking (Hiebert & Grouws, 2007).

Components of Number Sense

The National Council of Teachers of Mathematics (NCTM, 1989) identified five key components of number sense:

- **Number Meaning:** Understanding what numbers represent in various contexts. For example, a kindergartener recognizes that the number 3 can represent three apples, three friends, or three steps.
- **Number Relationships:** Recognizing how numbers relate to each other. For example, A first-grader understands that 5 can be composed with a 2 and a 3 or that 5 and 5 are the same as 6 and 4, which both compose 10.
- **Number Magnitude:** Grasping the relative size of numbers. For example, a second-grader can place numbers like 15, 8, and 23 in order on a number line, understanding that 23 is larger than both 15 and 8.
- **Operations Involving Numbers:** Understand how arithmetic operations affect numbers. For example, a first-grader realizes that adding 2 to 3 makes it larger (5) while taking 2 away from 5 makes it smaller (3).
- **Referents for Numbers and Quantities:** Using benchmarks or reference points for numbers and measurements. For example, a kindergartener uses their hand span as a reference to estimate the length of a crayon or understands that 10 is a benchmark for counting groups of objects.

Research Findings

These foundational skills contribute significantly to a child's mathematical development. The importance of number sense extends beyond basic numeracy. Research indicates that it is crucial for various mathematical competencies, including mental calculation, computational estimation, judging the relative magnitude of numbers, recognizing part-whole relationships and place value concepts, and problem-solving. These skills form the bedrock of mathematical thinking and are essential for success in more advanced mathematical concepts.

A significant study by Jordan et al. (2007) underscores the long-term impact of early number sense development. Their research found that early number sense skills predict later mathematics achievement, emphasizing the critical need to focus on these skills in the early years of education. This finding reinforces the idea that a strong foundation in number sense during the early years can have far-reaching effects on a student's mathematical journey, influencing their ability to grasp more complex concepts as they progress through their education.

Developing Number Sense

Key strategies for fostering number sense in young learners include:

- **Subitizing** is the ability to visually recognize small quantities without counting. This skill helps children move beyond one-to-one counting and see numbers as sets or groups (Clements, 1999). For example, show a child a dice with four dots for a brief moment and ask them to identify the number without counting each dot individually.
- **Counting with Understanding** goes beyond rote memorization and involves grasping the principles of one-to-one correspondence, cardinality, and counting forward and backward (Gelman & Gallistel, 1978). For example, children can count out five apples, touch each apple as they say the corresponding number, and then ask, "How many apples are there in total?" to reinforce the concept of cardinality.
- **Number Relationships:** Exploring how numbers relate to each other, including part-whole relationships and number comparisons (Van de Walle et al., 2013). For example, bar models can be used to show that 7 is composed of 5 and 2 or 3 less than 10.
- **Visual Representations:** Tools like number lines and bar models represent numbers and their relationships, which inherently highlight magnitude and proportions (Institute of Education Sciences, n.d.). For example, a number line showing that 8 is halfway between 6 and 10 helps children visualize number relationships.
- **Varied Arrangements:** Present objects in different configurations to prompt diverse mental strategies for recognizing quantities (Clements & Sarama, 2014). For example, eight objects can be shown arranged in a circle or a scattered pattern, but significantly and finally, in a line to help children recognize that the quantity remains the same regardless of arrangement.
- **Proportional Reasoning:** Introducing early concepts of proportionality and "sameness" to build a foundation for later mathematical understanding (Lamon, 2007). For example, a simple recipe where children need to double the ingredients, such as changing 2 cups of flour to 4 cups, introduces the concept of proportional relationships.

Teaching Approaches

Effective instruction for developing number sense should incorporate a variety of research-based approaches. Teachers should provide ample opportunities for hands-on exploration with concrete materials, allowing students to manipulate objects to understand number concepts physically. This tactile approach should be complemented by encouraging students to discuss and explain their strategies, fostering metacognition and deeper understanding. Utilizing a range of representations, from concrete objects to iconic models such as number lines and bar models, and then to abstract symbols, helps students build connections between different forms of mathematical thinking. These iconic models are particularly important for building lasting schemas, as they provide visual representations that bridge concrete experiences with abstract concepts.

Integrating number sense activities into daily routines and problem-solving contexts is crucial for making mathematics relevant and applicable. Teachers should focus on building flexibility in numerical thinking, helping students see or visualize numbers as malleable entities rather than fixed symbols. Number lines and bar models play a vital role in this process, allowing students to represent and manipulate numbers visually, reinforcing their understanding of number relationships and magnitude. As students progress, instruction should move from simpler concepts, such as working with numbers under 5, to more complex ideas involving numbers up to 10 and beyond. This gradual progression, coupled with consistent practice using varied representations, including iconic models and real-world applications, lays a solid foundation for advanced mathematical concepts in later grades.

Conclusion

In conclusion, developing number sense in early mathematics education is a critical foundation for students' long-term mathematical success. The five key components of number sense—number meaning, relationships, magnitude, operations, and referents—form the basis of a child's mathematical understanding. Research has shown that early number sense skills are strong predictors of later mathematics achievement, emphasizing the importance of focusing on these skills in the early years.

Effective strategies for fostering number sense include subitizing, counting with understanding, exploring number relationships, using visual representations, presenting varied arrangements, and introducing early concepts of proportional reasoning. These approaches help children develop a flexible and deep understanding of numbers and their relationships.

Educators should employ various research-based instructional methods to successfully implement these strategies. These include providing hands-on experiences with concrete materials, encouraging discussion and explanation of strategies, utilizing multiple representations (from enactive to iconic and symbolic), and integrating number sense activities into daily routines and problem-solving contexts. Using iconic models, such as number lines and bar models, is crucial in bridging concrete experiences with abstract concepts.

By focusing on developing strong number sense in the early years, educators lay the groundwork for students to grasp more complex mathematical concepts as they progress through their education. This solid foundation enhances students' immediate mathematical abilities and sets the stage for long-term success in mathematics and related fields.

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Social Media

Exciting research on developing number sense in students reveals powerful strategies for building a strong mathematical foundation in young learners. Our latest overview explores how key approaches like subitizing, visual representations, and early proportional reasoning contribute to a deeper understanding of numbers and their relationships. The research emphasizes that number sense is crucial for long-term mathematical success, with early skills predicting later achievement.

Visual representations, such as number lines, bar models, and hands-on activities, support number sense development by bridging concrete experiences with abstract concepts. This research bridges cognitive science with practical classroom applications, empowering educators to foster deeper mathematical understanding and problem-solving skills in their students. By implementing these evidence-based strategies, educators can lay a solid foundation for advanced mathematical concepts, setting students up for success. Join us in exploring these powerful learning strategies and their impact on early mathematical thinking!

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