

DMT INSIGHTS

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THE LEARNING TRIFECTA: HARNESSING STRUGGLE, MINIMAL HINTS, AND VARIED PRACTICE FOR LASTING MATH PROFICIENCY

Introduction

Mathematics education has long been guided by traditional practices that tend to promote immediate performance. However, research in cognitive psychology and education has revealed that some of these common practices may actually hinder long-term learning and flexible application of mathematical knowledge. This overview examines several counterintuitive approaches that, while challenging in the short term, lead to superior long-term outcomes in mathematical education. Research in mathematics education highlights several critical themes that influence student learning and long-term retention of mathematical concepts. A significant focus is the balance between immediate performance and enduring understanding and the instructional strategies educators employ (Hiebert & Grouws, 2007).

Embracing Productive Struggle

Traditional Approach: Many math curricula emphasize the rapid acquisition of procedural skills and algorithm memorization, aiming for quick mastery and improved test scores.

Counterintuitive Strategy: Focus on developing deep conceptual understanding through productive struggle, even if it means slower initial progress and potentially lower immediate test scores.

Research Findings: Studies have shown that teachers who excel at promoting contemporaneous student achievement may inadvertently harm their students' subsequent performance in more advanced classes. While students often give better evaluations to instructors who provide short-term benefits, emphasizing conceptual understanding through productive struggle leads to more durable and flexible knowledge that can be applied broadly (Kapur & Bielaczyc, 2012). The disconnect between short-term performance and long-term learning is a recurring theme in mathematics education research. Studies indicate that students and teachers often misinterpret immediate performance as a reliable learning indicator. For example, teachers who focused on promoting immediate student achievement were found to hinder students' long-term performance in advanced courses. This highlights the need for educators to adopt a more nuanced understanding of student progress, which values conceptual understanding over mere procedural fluency (Hiebert & Grouws, 2007). Hattie and Timperley's research on feedback and its impact on learning reinforces this point, indicating that students often misinterpret immediate performance as a reliable indicator of their

understanding (Hattie & Timperley, 2007). This underscores the importance of providing feedback that focuses on process and self-regulation rather than just task performance.

Implications for Practice:

- Design lessons and assessments that prioritize “making connections” problems, encouraging students to grapple with complex concepts.
- Implement project-based learning and real-world applications to deepen conceptual understanding through challenging, multi-step problems.
- Educate stakeholders about the long-term benefits of productive struggle, even if short-term progress appears slower.

Long-Term Benefits: While this approach may lead to slower apparent progress initially, it results in more robust mathematical understanding, improved problem-solving abilities, and better preparation for advanced mathematical concepts.

The Case Against Excessive Hint-Giving

Traditional Approach: Teachers often respond to student confusion by providing immediate hints or breaking down complex problems into simpler, procedural steps. This approach aims to reduce frustration and help students arrive at correct answers quickly.

Counterintuitive Strategy: Allow students to grapple with mathematical concepts without immediate resolution. Resist the urge to transform conceptual problems into procedural ones through excessive hint-giving.

Research Findings: Studies have shown that excessive hint-giving can impede long-term learning by transforming conceptual problems into procedural ones. Allowing students to struggle with problems enhances subsequent learning, even when initial answers are incorrect – a phenomenon known as the “hypercorrection effect.” Interestingly, being confidently wrong can lead to better retention of the correct information when learned later. Furthermore, the “generation effect” demonstrates that attempting to produce or generate an answer improves learning outcomes regardless of its correctness (Slamecka & Graf, 1978).

One of the key insights from studies is the detrimental effect of excessive hint-giving by teachers. In various classroom observations, it was noted that teachers often provide hints in response to student confusion, which can inadvertently transform conceptual problems into procedural ones. This tendency was particularly evident in U.S. classrooms, where approximately 20% of questions that began as conceptual problems lost their conceptual nature after teacher-student interactions. While this approach may yield short-term improvements in student performance, it undermines the development of a deeper understanding of mathematical concepts, ultimately affecting long-term learning outcomes (Kapur, 2014).

Cognitive load theory supports this observation, emphasizing that excessive hints can overload working memory, leading to superficial learning rather than deep understanding (Sweller, 1988). This aligns with the concept of desirable difficulties, which suggests that challenges that initially frustrate students can lead to more robust learning in the long run (Bjork & Bjork, 2011).

Implications for Practice:

- Design lessons that include “making connections” problems without immediately offering procedural guidance.
- When students ask for help, respond with guiding questions rather than direct hints.
- Encourage students to explain their reasoning and make connections between different ideas, even if their initial approach is incorrect.
- Create a classroom culture that values productive struggle and views mistakes as learning opportunities.

Long-Term Benefits: While this approach may lead to more errors and frustration in the short term, it fosters deeper conceptual understanding, improved problem-solving skills, and a greater ability to apply mathematical knowledge flexibly in novel situations.

Moving Beyond Blocked Learning

Traditional Approach: Math curricula and practice sessions often employ “blocked practice,” where students focus on one type of problem at a time. This method involves practicing a single skill repeatedly, allowing students to master one concept before moving on to the next (e.g., AAA BBB CCC).

Counterintuitive Strategy: Implement “interleaved” or varied practice, where different types of problems or concepts are mixed together during learning sessions. This approach involves alternating between various problem types, such as story problems, visual models, and symbolic algorithms (e.g., ABC ABC ABC).

Research Findings: Studies have consistently shown that while varied practice may feel more difficult and less effective in the short term, it leads to better long-term performance and knowledge transfer (Rohrer & Taylor, 2007). Varied practice improves inductive reasoning and the ability to apply knowledge flexibly. Students who engage in varied practice develop stronger abilities to recognize deep structural similarities across problem types (Kornell & Bjork, 2008).

Implications for Practice:

- Redesign homework assignments and in-class practice to include a mix of problem types rather than focusing on a single skill or idea.
- Create assessments that require students to discern problem types and apply appropriate strategies across various contexts.
- Revisit critical concepts throughout the year, gradually increasing complexity and mixing them with other topics.
- Explain to students and parents why this approach might feel more challenging but leads to better long-term outcomes.

Long-Term Benefits: Varied practice enhances students' ability to discriminate between problem types, select appropriate strategies, and apply their knowledge more flexibly in novel situations. This approach better prepares students for real-world problem-solving and advanced mathematical thinking.

Conclusion

These counterintuitive approaches – embracing productive struggle, minimizing hints, and implementing varied practice – challenge traditional methods in mathematics education. While they may lead to short-term difficulties and apparent setbacks, research consistently demonstrates their efficacy in promoting long-term learning, flexible knowledge application, and deeper mathematical understanding. By adopting these evidence-based practices, educators can foster the development of students who are not only proficient in mathematical procedures but also possess a deep, flexible understanding of mathematical concepts that can be applied across diverse contexts.

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Social Media

Exciting research on effective math learning strategies! Our latest overview explores the power of productive struggle, limited hint-giving, and varied practice in mathematics education. Discover how these counterintuitive approaches, while challenging in the short term, lead to superior long-term learning outcomes.

Key points: • Embracing productive struggle enhances deeper conceptual understanding • Limiting hints encourages students to develop problem-solving skills • Interleaved practice improves long-term retention and flexible application of knowledge • Prioritizing conceptual understanding over procedural fluency leads to more robust mathematical thinking

This research bridges cutting-edge cognitive science with practical classroom applications, empowering educators to foster deeper mathematical understanding and problem-solving skills. Join us in exploring these powerful learning strategies and their impact on mathematical thinking! #MathEducation #ProductiveStruggle #VariedPractice

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