

DMT INSIGHTS

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HARNESSING MISCONCEPTIONS: WHEN A GOOD IDEA IS USED IN THE WRONG PLACE

What students' misconceptions reveal about their mathematical thinking

Introduction

A misconception is more than a wrong answer. It develops when a student uses an idea or relationship that made sense in one situation in a new situation where it no longer applies. That is why many misconceptions make mathematical sense. A student who says 0.45 is greater than 0.7 may be reasoning that 45 is greater than 7. The comparison is correct for the whole numbers 45 and 7, but the student has carried that reasoning into decimals without attending to the unit. A misconception is harder to address than a simple mistake, because the answer came from an idea the student believes makes sense, and correcting the answer does not necessarily change that idea. This research overview integrates insights from mathematics education and cognitive psychology to examine how a misconception forms, why it persists, and how instruction can reorganize the idea that produced it (Borasi, 1987, 1994).

Key Idea • A misconception is often a reasonable idea used where it no longer applies.

Why a Misconception Persists

A repeated, systematic wrong answer may reveal a misconception: an idea that makes sense elsewhere but is being used where it no longer holds. Misconceptions of this kind persist even after direct teaching, because they arise from a student's own reasoning rather than from carelessness (Kapur, 2014; Borasi, 1987). Correcting the answer does not reach the idea underneath it, which is why the same misconception reappears.

A misconception shows how a student is thinking, making it a diagnostic tool rather than a defect to be removed. Building instruction around the misconception, rather than simply marking the answer wrong, leads to a deeper understanding of the mathematics (Borasi, 1994).

Key Idea • A misconception survives a corrected answer, because the idea that produced it is untouched.

Mistake or Misconception?

Mistakes and misconceptions call for different responses. In a mistake, the student understands the underlying idea but applies it incorrectly. A student who knows that 7 times 8 is 56 may write 54, and when asked to check it, will

usually correct it at once. In a misconception, the student reasons consistently from an idea that does not apply in the new situation. A student who says 0.45 is greater than 0.7 because 45 is greater than 7 may defend the answer when asked, because it follows from the idea they currently hold. A mistake usually needs correction. A misconception requires reorganizing an idea.

Key Idea • A mistake disappears when it is pointed out. A misconception persists because the idea still makes sense to the student.

Surfacing a Conception Before Instruction

Tasks that surface students' existing conceptions before formal instruction can be useful, because they give both the teacher and the student something specific to examine. Research on productive failure provides one example of how an initial attempt can prepare students for the instruction that follows (Kapur, 2014). The important point is not failure itself. It is making existing reasoning visible so instruction can act on it.

Key Idea • Make a student's existing reasoning visible, and instruction has something to act on.

Classroom Culture: Making a Misconception Visible

Misconceptions have to become visible before teachers can address them. Students therefore need opportunities to explain reasoning that may be incomplete or incorrect, without the discussion immediately ending with right or wrong (Alvidrez, 2019). The goal is not to celebrate wrong answers. The goal is to make the thinking available for examination.

Key Idea • A misconception has to be visible before it can be examined.

Four Misconceptions: A Good Idea in a New Situation

A misconception usually begins as an idea that worked. A student applies a familiar relationship to a new situation where it no longer holds (Supardi, 2021). Each of the following can be examined the same way. What idea is the student using? Where does that idea work? Where does it stop working? Moreover, what mathematical relationship does the student need to reorganize? The movement is not from wrong to right. It is from a conception that works in one place, to the boundary where it stops, to a conception reorganized for the new situation.

Example one moves from whole numbers to decimals. A student who says 0.45 is greater than 0.7 is often reasoning that 45 is greater than 7. That comparison is correct for whole numbers, which is where the idea comes from. It stops working in decimal notation, where 0.45 is forty-five hundredths, and 0.7 is seven tenths, or seventy hundredths. The relationship to reorganize is the unit: the question is what unit each number counts, not which string of digits is longer.

Example two moves from whole-number multiplication to multiplication by a fraction or decimal. Years of experience, where four times eight is greater than eight and six times twelve is greater than twelve, support the idea that multiplication makes a quantity larger. A student carries that idea into 8 times 0.5 and expects a result greater than 8. The idea holds whenever the multiplier is greater than one and fails when the multiplier is between zero and one. The relationship to reorganize is the multiplier's effect: multiplying a positive quantity by a number greater than one increases it, and multiplying by a positive number less than one decreases it.

Example three moves from whole-number addition to fractions. In whole-number addition, three plus four is seven, and the student combines the quantities being counted. Applied to one third plus one fourth, the same idea produces two sevenths. It works when the quantities are counted in the same unit and stops working when the units differ, because thirds and fourths are different-sized units. The relationship to reorganize is partition: both quantities must be expressed in a common unit, twelfths, before they can be composed, giving four twelfths and three twelfths, or seven twelfths.

Example four moves from whole-number place value to decimal place value. Students learn correctly that moving one place to the left makes a digit worth ten times as much. From this, some develop the incomplete idea that a digit farther to the right is simply a smaller number, without the multiplicative relationship between units. The idea is unfinished rather than wrong. The relationship to reorganize is that adjacent base-ten units are related multiplicatively: ten of one unit compose one of the unit immediately to its left, so one tenth is ten hundredths. This same structure governs whole-number place value, now extended across the decimal point.

Key Idea • In each case, a good idea is being used in the wrong place. Find where it holds, and you have found what to reorganize.

The Teacher's Role

Teachers guide students through surfacing and reorganizing a misconception. When a teacher treats a wrong answer as information about a student's reasoning rather than only as something to be corrected, students are more willing to show their thinking (Gunderson et al., 2011). Maintaining high expectations while helping students examine where an idea stops working supports both confidence and understanding.

Key Idea • Treat a wrong answer as information about reasoning, and students will show you their thinking.

Instructional Strategies: Examining the Error

Instructional strategies that examine students' errors can reach the misconception underneath them. Systematically investigating a student's mistake provides insight into the conception that produced it (Muhammad, 2021). Analyzing the error lets a teacher design a task that targets the idea itself rather than the surface answer.

Examining errors together promotes reflection and inquiry. When students explain their reasoning, they are prompted to think about where an idea works and where it does not, which strengthens mathematical reasoning. The move is to surface the idea, examine it, and reorganize it, not simply to replace the answer.

Key Idea • Surface the idea, examine it, reorganize it, rather than only replacing the answer.

Conclusion: Reorganize the Idea, Not Just the Answer

A misconception is different from a mistake. A mistake may disappear when you point it out. A misconception persists because the student is reasoning from an idea that makes sense to them. That makes misconceptions valuable instructional information. Ask what conception produced the answer. Find where that conception works. Then use a

model, a carefully chosen example, and precise mathematical language to show where it no longer holds and what relationship extends it. The goal is not simply to change the answer. It is to reorganize the idea underneath it.

Key Idea • **Change the idea underneath the answer, not just the answer.**

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0.45 or 0.7: which is larger? A student says **0.45**.

Ask why, and the reasoning becomes clear: **45 is greater than 7**.

That is not carelessness. The student is using an idea that worked with whole numbers—more digits can mean a larger number—and applying it to decimals, where that reasoning no longer holds.

Correcting the answer without addressing the reasoning leaves the misconception in place. The same thinking is likely to show up again.

A **misconception** is a reasonable idea used in a situation where it no longer fits. We do not just correct it. We help students reorganize their thinking: identify where the idea worked, show where it breaks down, and rebuild the idea for the new situation.

Inside this DMT Insight:

- Why misconceptions can survive a corrected answer
- How to distinguish a **mistake** from a **misconception**—and why they require different responses

- How to surface student reasoning before instruction so you know what you are building on
- Four common misconceptions and the reasonable ideas underneath them

Read the full Insight at mathsuccess.io/dmtinsights.

What misconception do you see every year—and what reasonable idea is underneath it?

Learn how collaborative approaches and productive struggle can help overcome misconceptions and build a stronger foundation for mathematical success!

