



Developing Mathematical Thinking:
What Depth of Knowledge Means in Math

DMTI INSIGHTS BY
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WHAT DEPTH OF KNOWLEDGE (DOK) MEANS IN MATH: RETHINKING WHAT WE ASK STUDENTS TO DO

(DMT INSIGHTS)

Introduction

Depth of Knowledge (DOK), developed by Webb (1997, 2002), is often described as a hierarchy of rigor. In practice, this misunderstanding quietly shapes classroom instruction. When rigor is equated with larger numbers, more steps, or longer assignments, lesson design drifts toward procedural intensity rather than conceptual depth. But DOK was never intended to measure how hard a problem feels. It was designed to classify the kind of thinking a task requires. **If we misunderstand that distinction, we unintentionally design lessons that train students to equate mathematics with speed and rote memorization rather than with structure and reasoning.**

A more precise way to understand DOK is as a taxonomy of mental actions. When students engage with a math task, what must they do cognitively? Must they retrieve a fact, interpret a context, construct a model, justify a claim, or analyze a pattern? **DOK describes the complexity of reasoning required, not the surface features of a task.** This shift is not theoretical — it directly affects how we design lessons and how we ask questions during instruction. The questions teachers pose determine the depth of thinking students practice.

From a cognitive psychology perspective, each DOK level engages different systems of thinking. Retrieval activates long-term memory; modeling requires integration of visual and symbolic representations; justification engages executive functioning and self-explanation (Sweller, 1988; Paivio, 1990). A task can be effortful — such as multi-digit multiplication with several carrying steps — without being cognitively complex. **True rigor comes from the reasoning demanded, not from the size of the numbers.**

DOK 1: Retrieval and Reproduction in Multiplication

DOK 1 tasks require recall or execution of well-practiced procedures. In multiplication, this might simply be computing 7×8 . The defining feature is reproduction without strategic decision-making. DOK 1 is foundational. Fluency reduces cognitive load and frees working memory for higher-order reasoning (Roediger & Karpicke, 2006). Students must move from consciously thinking about facts to thinking with them (Willingham, 2009). However, increasing the size of the numbers does not increase depth. A page filled with larger multiplication problems remains DOK 1 if students are simply executing a procedure. **Automaticity supports reasoning, but automaticity alone does not reveal conceptual understanding** (Hiebert & Carpenter, 1992). When lesson design emphasizes only DOK 1 tasks,

instruction centers on performance rather than structure.

DOK 2: Applying and Representing Mathematical Structure

DOK 2 tasks move beyond reproduction and require application or representation. In elementary mathematics, it is helpful to distinguish between two forms: contextual interpretation and conceptual modeling.

DOK 2A: Contextual Problem Solving and Quantitative Interpretation

A DOK 2A contextual task might state, “There are 7 rows of chairs with 8 chairs in each row. How many chairs are there?” Students must interpret the quantities, determine the relationship, and select multiplication as the operation. **The mental action is translating context into mathematical structure.** Research shows that difficulty with word problems often arises from the coordination of language and quantitative schemas (Kintsch & Greeno, 1985). Effective lesson design ensures that contextual tasks measure mathematical reasoning rather than reading complexity.

DOK 2B: Conceptual Modeling Through Representation

In DOK 2B, students represent mathematical structure using visual models. A task might ask students to draw an array or area model for 7×8 and explain what each dimension represents. Research on dual coding suggests that linking visual and symbolic forms strengthens understanding and supports transfer (Paivio, 1990; Fyfe et al., 2014). When students construct arrays or area models, they are building mental structures that later help them understand fractions, algebra, and proportional reasoning. Representing ideas visually makes mathematical structure visible. In this way, **DOK 2B supports relational understanding — knowing not just how to get an answer, but why the mathematics works** (Skemp, 1976). Teachers can deepen this level by asking students to compare different models, such as an array and a repeated addition model, to see how they both represent the same operation.

DOK 3: Strategic Thinking and Justification

DOK 3 tasks require strategic thinking and justification. In multiplication, students might be asked to explain why 7×8 equals 8×7 using a visual model, or to compare two strategies for solving 36×25 and determine which is more efficient. **The mental action shifts from doing mathematics to reasoning about mathematics.** When students justify, they articulate relationships using precise structural language — unit, compose, decompose, iterate, partition, and equal. Explanation strengthens conceptual networks and promotes durable learning (Chi et al., 1989). It transforms procedural knowledge into relational knowledge (Skemp, 1976). Instructionally, this requires teachers to pause and ask, “Why does that work?” or “How do you know the order does not change the product?” If such questions are absent, DOK 3 rarely occurs, regardless of how complex the numbers appear.

DOK 4: Extended Reasoning Across Conditions

DOK 4 tasks require sustained reasoning across multiple representations or conditions and push students beyond single problems into structured investigation. In multiplication, for example, students might generate all possible rectangular arrays with an area of 48 square units and analyze how the perimeter changes as factor pairs vary. This level demands synthesis and pattern analysis across cases; students must investigate systematically, make predictions, test conjectures, and draw conclusions from related data. Such work engages planning, monitoring, and generalization—forms of higher-order thinking identified as central to mathematical proficiency (National Research Council, 2001). Although extended reasoning tasks may not occur daily, they promote deep structural insight by requiring students to decide what to examine, how to organize their findings, and how relationships shift across conditions. In doing so, **DOK 4 moves learners from solving isolated problems to recognizing how mathematical ideas connect within a broader system.**

Implications for Educators and Instructional Practice

Educators sometimes wonder why children are asked to draw multiplication models instead of jumping straight to the standard algorithm. However, **representational competence—the ability to use and connect models—predicts long-term mathematical success (Clements & Sarama, 2014).** Drawing arrays for 7×8 builds mental structures that later support understanding of area models in algebra. Models are not detours from rigor; they are the *foundation* of rigor. From a cognitive science perspective, **these models give students a mental image to "anchor" the abstract symbol, making it more meaningful and memorable.**

Educators can support deeper thinking by asking students to explain their reasoning or show another representation. **Questions such as "How do you know?" or "Can you show that another way?" raise the cognitive demand without increasing stress.** Simple prompts can meaningfully increase the depth of thinking. For example, after a child computes 6×4 , an educator might say, "Show this using a bar model or an area model," or "If you forgot that fact, what is another way you could figure it out?" These questions gently move the child from DOK 1 toward DOK 2B (modeling) and DOK 3 (justification).

Clarifying Misconceptions About DOK

Several misconceptions about DOK persist. **Higher DOK does not mean larger numbers or longer problems.** Not all word problems are DOK 3; most are actually DOK 2A. DOK is not a staircase that students must climb step by step; it is a way to classify the demands of a task (Webb, 2002). **DOK describes the *nature of thinking*, not the order of instruction.** Furthermore, a task is not permanently "at" a certain DOK level. Its level depends on the thinking required of the student. **A task that asks for an explanation (DOK 3) becomes a DOK 1 task if the teacher has already provided and practiced the exact explanation.** Effective classrooms move flexibly among levels, reinforcing fluency while deepening conceptual reasoning. **Cognitive growth happens through this movement across levels, not by abandoning foundational skills.**

Conclusion

Depth of Knowledge provides educators with a precise tool for aligning instruction, cognition, and assessment. In multiplication and across all mathematical domains, **balanced cognitive demand supports conceptual understanding, procedural fluency, and strategic competence.** True mathematical rigor lies in the *quality of*

reasoning students are asked to perform. When instruction and assessments intentionally include retrieval, contextual interpretation, representation, and justification, **they reflect the full, multidimensional nature of mathematical proficiency (National Research Council, 2001)**. Through careful application of DOK, K–5 educators can design learning experiences that cultivate not only correct answers, but well-structured understanding—knowledge that lasts and transfers to new situations.

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Why do we keep confusing "hard" with "deep" in elementary math?

But that's not what Depth of Knowledge actually measures.

DOK isn't a staircase. It isn't a difficulty scale. It's a classification of the **mental actions** we ask students to perform.

Here's the uncomfortable truth:

A page of 4-digit multiplication problems is still DOK 1. But asking a student to explain — using an array — why 7×8 equals 8×7 ? That's DOK 2.

The depth is in the reasoning — not the numbers.

When lessons intentionally include:

- Retrieval (DOK 1)
- Context (DOK 2A)
- Modeling (DOK 2B)
- Justification (DOK 3)

Students don't just get faster. They get smarter about structure.

This isn't about making math harder. It's about designing questions that make thinking richer.

Take a look at your last lesson. What kind of thinking did it actually require?



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