



Developing Mathematical Thinking:
Geometric Thinking

DMTI INSIGHTS

BY

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THE "DIAMOND" PROBLEM: HOW EVERYDAY LANGUAGE UNDERMINES GEOMETRIC THINKING

(DMT INSIGHTS)

The Problem

A student points to a square drawn on a vertex and says, *"That's a diamond."* The poster on the wall agrees. The worksheet in the packet agrees too. In that moment, the student is not confused—they are being **perfectly consistent with the materials and language they have been given**. This is one problem of shape instruction: **students are often learning a *folk geometry***—a system of picture-based categories grounded in orientation, symmetry, and cultural labels—rather than the mathematics of invariant properties.

Search "triangle definition for kids." Browse shape posters from major curriculum publishers. Scan popular geometry activities online. Again and again, definitions and images emphasize **what shapes look like** instead of **what must be true**. These materials do not merely fail to correct misconceptions; they actively **train them** (Dağlı & Halat, 2016; Verdine et al., 2016). **This overview argues that repairing shape instruction requires reorganizing it around a single, coherent spine:** viewing shapes as **systems of intersecting straight lines** that form closed structures, from which vertices, sides, and properties emerge in a logical order (Lehrer & Schauble, 2015).

Introduction: The Core Cognitive Shift

Why do so many students—and adults—believe a shape can change simply because it is turned?

Polygons are one of the earliest places in mathematics where learners must move from **"it looks like..."** to **"it must have..."** from **"pictures"** to **"properties"**. This shift—from perceptual familiarity to definition-based reasoning—is not cosmetic. It underlies later learning about **transformations, area, similarity, congruence, and proof** (Lehrer & Schauble, 2015).

Research from mathematics education and cognitive psychology converges on a shared explanation: **students construct shape categories from the examples, language, and images they encounter most often**, not from formal definitions alone (Duval, 2006; Verdine et al., 2016). When those examples are narrow and prototype-heavy, categories become fragile and orientation-dependent.

Effective shape instruction therefore, requires three integrated forms of knowledge:

- **Mathematical structure** (definitions as constraints, not pictures),
- **Cognitive development** (how concept images form and narrow), and
- **Language awareness** (how everyday terms like *diamond* create competing taxonomies).

A unifying instructional stance is to adopt the **intersecting-lines lens**: treating polygons not as finished pictures, but as structures that emerge when straight lines intersect and close in space.

A Foundational Reframe: Shapes as Lines That Intersect in Space

What if students learned shapes as structures that emerge when lines intersect?

Most shape instruction begins with completed figures—a triangle drawn upright, a square resting on a side, a “diamond” tilted on a vertex. Cognitively, this privileges the final image and invites classification by appearance. The intersecting-lines lens reverses that order.

In this framing, shapes are **not objects**, but **outcomes of relationships**. When straight lines intersect, they create **points of intersection**. When those intersections are connected by straight segments, they form **edges**. When those edges connect in a closed chain, a **polygon** is created. The named shape—triangle, quadrilateral, pentagon—is the result of these constraints, not the starting point.

This view aligns with research showing that geometric understanding deepens when learners attend to **relations, constructions, and transformations**, rather than static images (Battista, 2007; Duval, 2006). Lehrer and Schauble (2015) emphasize that learning progressions depend on helping students coordinate representations and relationships—seeing figures as systems that can be composed, decomposed, and reorganized.

From this perspective, **vertices are not “corners” of a picture**, but **points where line segments intersect**. This distinction matters. When vertices are defined relationally, students are better able to identify them in concave figures, extremely acute or obtuse angles, and non-prototypical shapes. Research shows that vague “corner” language contributes to systematic errors in vertex counting and shape classification (Duval, 2006; Dağlı & Halat, 2016).

Once intersections are established, attention turns to the **segments between them**, which must be **straight**, not curved. Emphasizing straightness early helps students distinguish polygons from curved figures such as circles—distinctions often blurred by definitions that describe polygons merely as “flat shapes” (Verdine et al., 2016).

Next comes **closure**. A polygon exists only when straight segments connect in a closed loop with no gaps. Studies show that when closure is not treated as a necessary condition, students routinely accept “almost closed” figures as legitimate shapes (Dağlı & Halat, 2016).

Only after straightness and closure are secured does it make sense to attend to the **number of vertices (and sides)**. Counting vertices as intersection points naturally leads to classifying polygons by number—three for triangles, four

for quadrilaterals, five for pentagons, and so on—revealing these shapes as members of a single family differentiated by a structural parameter (Verdine et al., 2016).

Finally, **properties**—equal side lengths, right angles, parallel sides—become tools for refinement rather than sources of confusion. Properties no longer decide whether a figure is a polygon; they decide **which polygon it is**. This ordering supports inclusive hierarchies and reduces reliance on visual prototypes (Fujita & Jones, 2007).

The Mathematical Lens: Constraints, Hierarchies, and Structure

What must be true—regardless of orientation—for a figure to belong to a polygon category?

Mathematically, polygons are **simple, closed plane figures composed of straight line segments**, classified by number of sides and refined by properties such as angle measures, parallelism, and symmetry (NGA & CCSSO, 2010). The intersecting-lines lens makes these constraints explicit and inspectable.

This lens provides a productive alternative to appearance-based disagreement. When a student claims a rotated triangle is “not a triangle,” the teacher can redirect attention to invariants: number of intersections, straight segments, and closure—rather than debating orientation.

Quadrilaterals exemplify how mathematical categories are built by **adding constraints**. Parallelograms require parallel sides; rectangles add right angles; rhombi add equal sides; squares satisfy both constraints simultaneously (Fujita & Jones, 2007). **This inclusive hierarchy is mathematically elegant but cognitively difficult**, because everyday categories are often exclusive. Understanding this tension is central to teachers’ Mathematical Knowledge for Teaching (MKT).

The Cognitive Hurdle: Prototypes and Concept Images

Why does changing a shape’s orientation disrupt classification even when no defining property changes?

Cognitive research shows that children initially organize shape categories around **canonical exemplars**—upright triangles, squares resting on a side, highly regular polygons (Verdine et al., 2016). These prototypes support quick recognition but **narrow the category**. Duval (2006) characterizes this as a tension between **concept image** (what the category “looks like”) and **concept definition** (what properties determine membership). When orientation changes, the visual match fails—even though the definition still applies. **Robust understanding develops only when learners encounter systematic variation** in orientation, size, and regularity. Variation is not enrichment; it is the mechanism by which definitions gain meaning (Verdine et al., 2016).

Applying the Intersecting-Lines Lens to Persistent Problem Areas

When students make predictable shape errors, what rule are they using—and how can instruction replace it?

Triangles: The “Skinny Triangle” Rule

Many students accept only upright, isosceles triangles and reject obtuse or rotated examples (Dağlı & Halat, 2016). **The implicit rule is appearance-based. Instructional move:** Return to invariants—three straight sides, three vertices, closed—and ask what changed and what stayed the same under rotation (Lehrer & Schauble, 2015).

Quadrilaterals: The Hierarchy Conflict

Students resist “a square is a rectangle” because they treat categories as exclusive. **Distinct names imply distinct kinds** (Fujita & Jones, 2007). **Instructional move:** Re-center on defining attributes (four right angles) and use nested diagrams or property grids to make inclusion visible.

The “Diamond” Problem: Language as Taxonomy

Diamond is not a mathematical category, yet its use introduces a **parallel, orientation-based taxonomy** (Duval, 2006; Verdine et al., 2016). **Instructional move:** Use *diamond* as a contrast—everyday word vs mathematical classification—and verify invariants explicitly.

The Tyranny of “Nice” Shapes

Irregular pentagons and hexagons are often rejected as “not real,” especially when curricula overrepresent regular examples (Verdine et al., 2016). **Instructional move:** Design example spaces that force property-checking rather than aesthetic judgment.

The Instructional Path Forward: From Naming to Structural Reasoning

What daily routines make property-based reasoning inevitable rather than optional?

Research describes a progression from **visual recognition** to property analysis to **relational reasoning about classes and hierarchies** (van Hiele, 1986). This progression aligns with standards across K–8 (NGA & CCSSO, 2010).

Instruction strengthens when teachers model structural actions—compose, decompose, partition, and reason about equal sides and angles—rather than relying solely on identification (Clements & Sarama, 2011). These actions connect early shape work to later ideas in area, similarity, and proof.

A practical routine across grades is the **3-check method**:

1. **Straight** sides?
2. **Closed** figure?
3. **Count** vertices/sides,
4. **Apply** properties.

Conclusion: Choosing a Geometry of Pictures or Properties

If students learned geometry only from your examples and language, what theory of “what makes a shape a shape” would they construct?

Every classroom teaches either a **geometry of pictures** or a **geometry of properties**. That choice is made not in standards documents, but in **posters, examples, definitions, and daily language**. **The same mechanism that creates misconceptions can eliminate them:** change the training set. Broaden examples, reorder definitions, make hierarchy visible, and treat everyday labels as contrasts—not categories (Duval, 2006; Fujita & Jones, 2007; Lehrer & Schauble, 2015; Verdine et al., 2016).

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Social Media

Why do so many students think a square turns into a “diamond” when you rotate it?

It is not a student problem—it is a materials problem.

Our latest **DMT Insight, *The Diamond Problem: How Everyday Language Undermines Geometry Reasoning*** shows how common posters, worksheets, and online resources quietly teach a *picture-based* geometry:

- Rectangles are “the long ones”
- Triangles must look upright
- Rotation changes what a shape *is*

Drawing on research from mathematics education and cognitive psychology, this overview argues for a simple but powerful reframe:

Teach shapes as systems of intersecting straight lines, not pictures to recognize.

When students focus on **intersections, straightness, closure, number of vertices, and then properties**, misconceptions like “diamond” fade—and durable geometric reasoning takes hold.

If you work with K–8 teachers, instructional coaches, or curriculum teams, this is a must-read.

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