



Developing Mathematical Thinking:
Beyond Name the Shape

DMTI INSIGHTS BY
Jonathan Brendefur, PhD
and Associates

BEYOND “NAME THE SHAPE”: TEACHING QUADRILATERALS FOR UNDERSTANDING

(DMT INSIGHTS)

Introduction

Why do so many students insist that a square cannot be a rectangle — and what does that tell us about how we teach quadrilaterals?

These are not random errors. They are predictable outcomes of how quadrilateral instruction is typically designed. In most classrooms, learning about quadrilaterals begins and ends with naming shapes—matching words to pictures. Students learn to recognize what shapes look like, not what must be true about them. Rarely do they investigate what must always be true about a shape, regardless of how it is oriented or drawn.

This matters because **quadrilaterals sit at a critical crossroads in mathematics education**. They are one of the first sustained opportunities students have to reason about attributes, classification systems, and logical relationships. When instruction develops that reasoning, quadrilaterals become a foundation for algebraic thinking, proportional reasoning, and eventually proof. When instruction reduces them to vocabulary, students build fragile, picture-based categories that collapse the moment a familiar shape is rotated or presented in an unfamiliar form.

Research in mathematics education and cognitive psychology points to the same conclusion: **durable geometric understanding begins when students learn to see shapes not as finished pictures**, but as structures defined by relationships among lines (Hiebert & Carpenter, 1992; Lehrer & Schauble, 2015). This DMT Insight examines what that shift requires, why students struggle without it, and what a more effective instructional path looks like.

The Problem: Picture-Based Geometry

What happens when students learn shapes as pictures rather than as structures defined by properties?

When geometry instruction centers on memorizing the appearance of shapes, students develop what researchers call prototype-based reasoning. They form mental templates from repeated examples—the upright triangle, the horizontal rectangle, the square resting flat on a side—and judge whether a shape belongs to a category by how closely it resembles that template (Rosch, 1978). **The result is a geometry of appearances rather than a geometry of properties.**

The consequences are well-documented. Students routinely:

- Reject a rotated square as “not a real square” or label it a diamond
- Insist that a square is not a rectangle because rectangles are “the long ones.”
- Fail to recognize non-prototypical examples—an irregular quadrilateral and a non-isosceles trapezoid
- Accept shapes as belonging to a category based on overall look rather than defining properties

These errors are not signs of inattention. They are the logical result of instruction that emphasizes what shapes look like over what must be true about them. Clements and Battista (1992) found that students frequently believe a square is not a rectangle because they interpret category labels as mutually exclusive. De Villiers (1994) showed that **failure to teach hierarchical inclusion**—the idea that one category can be a subset of another—**undermines students’ capacity for geometric reasoning well into middle school.**

The same pattern shows up in everyday classroom materials. Worksheets show parallelograms only leaning to the right. Definitions describe rectangles as “longer than they are tall.” **These materials do not merely fail to correct misconceptions—they actively train them** (Dagli & Halat, 2016; Verdine et al., 2016).

The Cognitive Foundations: Why This Is Hard

What does cognitive psychology tell us about why quadrilateral learning is particularly challenging?

Cognitive research offers a clear explanation for why these misconceptions are so persistent. Two phenomena are especially relevant: prototype effects and the distinction between concept image and concept definition.

Prototype effects describe the well-established tendency **to judge category membership by similarity to a typical example** rather than by formal criteria (Rosch, 1978). In geometry classrooms, the typical rectangle is horizontal and longer than it is tall; the typical square sits flat. When students encounter atypical examples—a tall, narrow rectangle, a rotated square—they often reject them as non-members, not because they lack the defining properties, but because they do not match the mental template.

The concept image/concept definition distinction, developed by Vinner and Hershkowitz, helps explain why this problem persists even after formal instruction. **Concept image refers to everything a student mentally associates with a concept**—all the visual impressions, prior examples, and informal rules accumulated over time. **Concept definition is the formal, precise description of what makes something a member of the category.** Students may be able to recite a definition correctly while their concept image—built from years of prototype-heavy exposure—continues to drive their actual classification decisions (Vinner, 1991).

Working memory research adds another layer. When a student encounters a new shape, the visual stimulus automatically activates the concept image system through fast, pattern-matching recognition that requires minimal cognitive effort. The formal definition, by contrast, must be deliberately retrieved and applied—a slower, more effortful process. Under typical classroom conditions, the automatic system wins. Students say “that’s a rectangle” before they have a chance to check whether the properties match (Sweller, 1988).

This is why varied exposure matters so much. **Variation in orientation, size, and regularity is not enrichment—it is the mechanism by which formal definitions gain genuine meaning.** Without it, definitions remain inert, and concept images continue to govern reasoning.

The Mathematical Structure: Lines, Properties, and Hierarchies

What does it mean to understand quadrilaterals as structures defined by line relationships rather than as pictures to name?

A more powerful instructional foundation begins not with shape names, but with lines. Every quadrilateral is, at its core, the bounded region formed when four line segments connect in a closed path. The properties that differentiate quadrilateral types—parallelism, perpendicularity, and congruence of sides and angles—are relationships among those lines.

From this perspective, a parallelogram is the region bounded by two pairs of parallel lines. A rectangle is a parallelogram whose lines intersect at right angles. A rhombus is a parallelogram whose paired lines are equally spaced. A square satisfies both conditions simultaneously. **Understood this way, the hierarchical structure of quadrilateral families becomes logical rather than arbitrary: all squares are rectangles because every square satisfies all the conditions for a rectangle and then some.** The category is inclusive, not exclusive.

This line-intersection foundation is mathematically important for several reasons. It makes orientation irrelevant—the relationship between two parallel lines does not change when the figure is rotated. It connects shape categories to their defining constraints rather than their typical appearances. And it provides a bridge between a student’s visual experience and formal mathematical reasoning: a student can look at a rotated rectangle and ask, “Are these opposite lines parallel? Do these adjacent lines form right angles?” rather than asking, “Does this look right?”

Research confirms that students who understand shapes through their underlying structural properties are better equipped to reason about class inclusion, to recognize shapes across orientations, and to transfer that thinking to novel problems (Fujita & Jones, 2007; Clements & Sarama, 2009).

What Research Tells Us About Developmental Progressions

How does geometric thinking develop, and what does this mean for instruction?

Research on geometric thinking describes a progression from visual recognition to property-based analysis to relational reasoning about hierarchies and inclusion. At early levels, students identify shapes based on overall resemblance. At intermediate levels, they attend to specific attributes such as the number of sides or the presence of right angles. Only at more advanced levels do they reason about logical relationships—understanding why one category can be a subset of another, or why a single shape can simultaneously belong to multiple categories (Clements & Battista, 1992).

Underlying this progression is an understanding of lines themselves. A line is not a segment drawn on paper—it is an infinite object, extending in both directions without end. What students see in a shape are traces of lines: the sides of a quadrilateral are segments, but they lie on lines that continue beyond the figure. Parallel lines never meet, no matter how far extended; perpendicular lines meet at exactly a right angle. **When students grasp lines as objects**

with these properties in space—not merely as visible edges—they gain the conceptual foundation for understanding why quadrilateral categories exist and how they relate to one another.

This progression does not happen automatically. Research consistently shows that without explicit instruction designed to move students from visual to relational reasoning, students remain at descriptive levels well into middle school. Simply presenting definitions does not produce conceptual reorganization. What moves students forward is a carefully designed instructional experience: **exposure to varied examples and non-examples, sorting tasks that require justification, and opportunities to reason explicitly about what properties define a category and why some shapes qualify for more than one.**

The cognitive challenge at the relational level is significant. Understanding that all squares are rectangles requires grasping that the set of squares is a proper subset of the set of rectangles—that satisfying more constraints is a stronger condition, not a different one. This logical structure is cognitively demanding, particularly for students who have been taught to treat shape names as mutually exclusive labels. It develops gradually and requires instruction that makes the logic visible, not just the vocabulary.

Moving Forward: What Effective Instruction Looks Like

What are the highest-leverage shifts educators can make?

The research points to several instructional principles that can shift quadrilateral learning from picture-based memorization to property-based reasoning. **Effective instruction begins with lines rather than shape names**, prioritizes the question "What must be true?" over "What does this look like?", and uses varied, systematic example spaces—including rotated, stretched, and non-prototypical examples alongside carefully chosen non-examples. Making hierarchical relationships explicit through nested diagrams helps students internalize inclusive logic rather than defaulting to mutually exclusive thinking. Crucially, strong formative assessment reveals student reasoning, not just answers: questions like "What would have to change for this to become a square?" distinguish genuine understanding from pattern-matching. For specific teacher moves and classroom strategies that bring these principles to life, see our *Quadrilateral Companion Document*. *Note: for a classroom-ready visual reference, see the DMTI Quadrilateral Poster, available on Teachers Pay Teachers and Redbubble.*

Conclusion

The persistent difficulties students experience with quadrilaterals—the square-rectangle confusion, the orientation errors—are not mysteries. They are the predictable outcomes of instruction that teaches shapes as pictures rather than as structures defined by invariant properties.

When instruction is redesigned around line relationships, property-based classification, and explicit hierarchical reasoning, quadrilaterals become something far more powerful than a vocabulary unit. They become grounds for logical thinking, for the discipline of asking "What must be true?" rather than "What does this look like?"—and for the intellectual habits that will serve students across every domain of mathematics. **Properties over Pictures.**

References

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Social Media

Why do so many students insist that a square is not a rectangle — and why does that one misconception matter so much?

Because geometry instruction too often teaches shapes as pictures to memorize rather than structures defined by properties. Our latest DMT Insight shows:

- Student misconceptions about quadrilaterals are predictable—and preventable
- Prototype-based reasoning narrows how students classify shapes
- Teaching shapes as line-intersection structures eliminates orientation errors
- Hierarchical reasoning (all squares are rectangles) requires explicit instruction, not just definitions

This isn't a student problem. It's an instruction problem.

Note: a classroom-ready quadrilateral poster is available on TPT and Redbubble.

What's one misconception about quadrilaterals you see every year? Share your experience at contact@dmtinstitute.com



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