DMTI Math Success

Developing Mathematical Thinking:
Beyond Worksheets: How Summer Math Programs Boost Language and Thinking

RESEARCH OVERVIEW

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BEYOND WORKSHEETS: HOW SUMMER MATH PROGRAMS BOOST LANGUAGE AND THINKING

(DMT INSIGHT)

Introduction

Summer math programs often focus on skill practice, but true mathematical understanding requires more: critical thinking, precise language, and the ability to explain reasoning. **This research explores how effective summer math programs go *beyond* worksheets to foster deep, lasting mathematical growth.** We investigate the relationship between rich early math experiences (incorporating storytelling, problem-solving, and mathematical discourse) and improved student outcomes, including stronger academic performance and enhanced language skills. This overview addresses key questions for educators designing impactful summer programs: How can we cultivate flexible mathematical understanding? What progression of models and language best supports this growth? When should we encourage student-generated strategies versus formal methods? The answers provide practical guidance for creating engaging and effective summer math initiatives.

From Math Stories to Academic Success

How do early math experiences in summer programs shape students' future academic achievement?

Research consistently shows that **early mathematics skills are foundational for later math learning and are among the strongest predictors of later reading and overall academic success,** often surpassing the predictive power of early reading skills (Duncan et al., 2007; Nguyen et al., 2021). This strong correlation stems from the development of crucial cognitive and linguistic skills through early math experiences. Executive functions—such as working memory and cognitive flexibility—are significantly enhanced by engaging with mathematical concepts, as is the acquisition of precise mathematical language. Consequently, these skills are transferable and support both mathematical and literacy development. These findings underscore the importance of early, intentional math experiences in shaping students' long-term academic trajectories. Effectively designed summer programs significantly strengthen students' cognitive and linguistic skills by incorporating activities focused on math storytelling, rich problem-solving tasks, and language-rich routines, setting them up for greater success in subsequent academic years.

Evidence: Impact on Learning and Engagement

How do summer math programs influence students' academic growth and engagement?

Recent research demonstrates that **summer math programs have a significant, positive impact on student learning and engagement**. A comprehensive meta-analysis of 37 experimental and quasi-experimental studies found that students who participated in summer math programs achieved notably higher mathematics outcomes than their peers, with an average effect size of +0.10 standard deviations, which represents a meaningful improvement in math scores (Lynch, An, & Mancenido, 2022). These results highlight that the benefits of summer math programs are not limited to academic achievement but extend to fostering positive attitudes toward mathematics and school participation. **These gains are consistent across both higher- and lower-poverty settings**, highlighting the equity potential of well-designed summer programs.

This positive impact on engagement is directly linked to program design, as studies show improvements particularly in programs that prioritize hands-on activities, collaborative learning, and rich mathematical discourse (Lynch et al., 2022). These findings strongly support the implementation of active, student-centered summer math programs as an effective strategy for improving both academic achievement and student engagement in mathematics.

Persistent Challenges and Their Solutions

Why do some students struggle to make gains in traditional summer math programs?

A common flaw in many summer math programs is their emphasis on remediation and rote skill practice, which often limits student engagement and deeper understanding. This emphasis on speed and memorization, often reflected in program design, perpetuates persistent misconceptions—such as viewing math as a set of isolated procedures—and hinders the development of deeper understanding. Furthermore, **when foundational topics like measurement, spatial reasoning, problem-solving, and the use of visual models are neglected, students miss out on essential building blocks for later mathematical achievement.** Research shows that early mastery of these areas strongly predicts later success in mathematics and reading (Duncan et al., 2007; Verdine et al., 2017; Mix & Cheng, 2012). Programs that integrate math with literacy, through storytelling, collaborative problem-solving, and culturally relevant contexts, yield greater gains in both mathematics and reading comprehension scores than literacy-only approaches. **Embedding mathematics within literacy-rich environments accelerates math learning and enhances literacy.** Addressing these persistent challenges requires intentional program design that values depth over speed and provides equitable access to rich mathematical experiences for all students.

An Instructional Models

What happens when students use real-world models and stories before formalizing mathematical procedures?

Effective summer math programs, such as the DMTI Summer Program, utilize a carefully sequenced progression of learning experiences, moving from concrete, hands-on activities (e.g., using story mats and manipulatives to solve word problems involving addition and subtraction) to visual models (e.g., bar models and number lines to represent problem situations) and finally to symbolic representations and equations. Manipulatives foster deep conceptual understanding, while visual models seamlessly bridge the gap between the concrete and the abstract, allowing students to connect their actions to symbolic notation and build lasting understanding. Each week's curriculum is

structured around rich problem-solving activities that are framed within culturally relevant contexts, emphasizing language development, strategic formalization, and varied practice, including both independent and collaborative work. This approach ensures students connect mathematical concepts to language, culture, and real-life contexts. This enactive-iconic-symbolic approach, grounded in well-established learning theories like Bruner's three modes of representation, builds a strong foundation for future learning (DMTI, 2025).

Integrating Mathematical Language and Literacy

How can integrating mathematical language and literacy strategies in summer math programs improve both students' conceptual understanding and their overall academic achievement?

Integrating mathematical language and literacy practices is essential for fostering deep conceptual understanding and academic growth in summer math programs. Research demonstrates that when teachers explicitly emphasize precise mathematical vocabulary (such as unit, compose, decompose, iterate, partition, and equal), facilitate structured mathematical discourse, and incorporate writing activities like math journaling and story creation, students' academic language proficiency is significantly strengthened—a critical foundation for mathematical reasoning and comprehension (Lynch et al., 2022; Reynolds & Yavuz, 2022). **Embedding these language-rich strategies within problem-based learning and literacy activities**—such as story mats, read-alouds, and collaborative discussions—not only supports comprehension and mathematical modeling (Lenhoff et al., 2020) but also **increases engagement and leads to improved mathematics achievement.** By connecting math problems to students' local contexts and encouraging reflection through both writing and oral explanation, summer programs enhance student learning, promote long-term retention, and bridge the gap between math and literacy skills. This integrated approach supports not only higher mathematical achievement but also broader academic success, particularly for students from diverse backgrounds (Lynch et al., 2022; Reynolds & Yavuz, 2022). This interaction between language and mathematics not only supports achievement in both domains but also prepares students for the increasingly interdisciplinary demands of future learning.

Conclusion: The Evidence is Clear

How can summer programs become launchpads for reasoning and confidence?

Research overwhelmingly demonstrates that **effective summer math programs prioritize conceptual understanding, integrated literacy, and foundational skills**—moving far beyond rote practice and remediation. By centering instruction on mathematical discourse, precise vocabulary, and writing activities, and by connecting learning to meaningful, real-world contexts, school districts can create engaging environments where students build both academic skills and confidence. Incorporating visual and symbolic models, as well as opportunities for reflection through journaling and oral explanations, ensures that learning is both meaningful and enduring. **Ongoing professional development and collaborative planning among educators further maximize program impact.** Thoughtfully designed, research-driven summer math experiences can truly transform students' academic trajectories, supporting every child's growth, confidence, and future success.

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Social Media

Maximize Summer Learning: Boost Math & Literacy Skills Simultaneously

Invest in high-impact summer math programs that deliver a double return: improved math *and* literacy skills. Research shows these programs significantly enhance students' academic language proficiency, mathematical reasoning, and overall confidence. By incorporating evidence-based strategies like precise vocabulary instruction, collaborative discussions, and hands-on activities, you can equip your students for success in the upcoming school year. **Effective summer programs reduce the summer slide, improve test scores, and create a more engaging learning environment.** This represents a highly efficient use of resources, maximizing student outcomes.

Learn more about evidence-based summer learning strategies: www.dmtinstitute.com

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