

DMT INSIGHTS

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PLACE VALUE: THE UNDERSTANDINGS THAT PREDICT LATER MATH SUCCESS

Introduction: Place Value as Units and Magnitude

By third grade, most students can name the places in a multi-digit number. They can point to the tens place in 347 and the hundreds place in 5,026. **Naming positions, however, is not the same as understanding them**, and the gap between the two becomes consequential in grades 3–6, where the mathematics begins to demand that students reason about the quantities the digits represent. What matters in these grades is whether the student understands what units the digits represent, how those units are composed, and where the number sits in the broader number system (Fuson, 1990).

Place value is the structure of the base-ten system. It allows any whole number — and, later, any decimal — to be written with ten digits because **the value of each digit depends on its position**. Understanding that structure requires coordinating three ideas at once: a digit tells how many units there are, the place tells what kind of unit is being counted, and the number as a whole names a single quantity. In 526, the 5 is not “five” but five units of hundred, the 2 is two units of ten, and 526 is one quantity rather than three separate digits. Coordinating these ideas is a developmental achievement that takes years to build (Ross, 1989).

This overview treats place value as two intertwined understandings — **units, or how quantities are composed and decomposed, and magnitude, or how large a number is and its location** — and frames them for formative assessment. For each developmental understanding, it describes what the understanding is, why it predicts later achievement, and the kinds of items that reveal whether a student holds it. The aim is to give teachers in grades 3–6 a way to diagnose place-value reasoning, not just confirm that students can label positions.

Key Idea • Place value is the ability to reason about the units a digit represents and the magnitude of the number — two understandings that go well beyond naming positions.

Why Place Value Predicts Later Achievement

Place-value understanding is not a topic completed in the primary grades; it is a way of reasoning that the mathematics of grades 3–8 continues to draw on. **Multi-digit addition and subtraction require decomposing and recomposing units** — computing $503 - 178$, for instance, means partitioning one unit of hundred into ten units of ten. Multiplication and division require working with composite units. Fractions and decimals require nested units and magnitude. Measurement requires iterating units, and algebra requires seeing numbers and expressions as composed quantities. When place value is shallow, students can often succeed on simple cases while failing as numbers grow, as regrouping is required, or as decimals, fractions, and powers of ten enter (National Research Council, 2001).

The predictive evidence is strong and specific. In a longitudinal study following students from first through fifth grade, **the accuracy of children's number-line placements** — a direct measure of magnitude understanding — **uniquely predicted mathematics achievement** and its growth, even after controlling for intelligence, working memory, and processing speed (Geary, 2011). A meta-analysis of 263 effect sizes from more than 10,000 students found that number-line estimation correlates moderately and consistently with broader mathematical competence, with the relationship strengthening as students move into fractions (Schneider et al., 2018). In nationally representative samples from the United States and the United Kingdom, elementary students' knowledge of fractions and division, both of which rest on place-value and magnitude understanding, uniquely predicted algebra and overall mathematics achievement in high school five to six years later (Siegler et al., 2012).

A student who sees 4,000 as 40 hundreds can divide it by ten at a glance; one who knows only that the 4 is in the thousands place cannot. That is what makes place value a high-value target for formative assessment: **the same reasoning a teacher can detect in grade 3 continues to pay off through algebra** (Booth & Siegler, 2008).

Key Idea • Because unit structure and magnitude are what predict later achievement, assessing them early — while there is still time to act — is worth the effort.

Unitizing: The Foundational Predictor

The strongest predictor of place-value understanding is **unitizing** — **the ability to treat a group as a single unit while still knowing what it is made of**. A student who sees ten ones only as ten separate objects is reasoning additively; a student who can also see those ten units of one as one unit of ten has begun to unitize. Unitizing is what allows students to work with composite units: a hundred is ten tens or one hundred ones, and a thousand is ten hundreds or one hundred tens. This nesting of units is the structure of the base-ten system (Fuson, 1990; Lamon, 2007).

In grades 3–6, unitizing is the understanding that lets students rename numbers fluidly for computation — **seeing 340 as 34 tens** when dividing by ten, or 4,000 as 40 hundreds when reasoning about scale. Without it, students may name places correctly while still treating the 4 in 47 as a 4 rather than as 4 tens, which limits their ability to reason

about regrouping, multiplication by powers of ten, and decimals (Ross, 1989).

Items that ask students to count in composite units expose whether unitizing is in place: *How many tens are in 340? How many hundreds are in 2,500? I have 47 tens — what number is that?* A student who must build up by ones, or who answers “4 tens” for 340 by reading only the tens digit, is naming positions rather than reasoning about units. The diagnostic value lies in the explanation: ask how the student knows, and listen for whether they coordinate the unit (tens) with the count (34).

Key Idea • Unitizing — treating a group as one unit while knowing its composition — is the foundational place-value understanding; assess it by asking how many tens or hundreds are in a number and listening to the reasoning, not just the answer.

Magnitude and Number Sense: Locating Numbers, Not Just Writing Them

Place value also depends on magnitude — understanding how large a number is and where it falls relative to others, beyond how it is written. Students with magnitude understanding know that 80 is much closer to 100 than to 10, that 503 is just over 500, and that 2,000 is ten times 200. A place-value chart shows how a number is composed; a number line shows where it lives. Students need both, and the number line is the representation that most directly externalizes magnitude (Booth & Siegler, 2008).

Magnitude understanding is what **keeps students from treating digits independently**. A student attending to the quantity recognizes that 402 is greater than 389 even though 389 has larger digits in the tens and ones places. As the longitudinal and meta-analytic evidence shows, the precision of students’ number-line placements is among the most robust predictors of later achievement, which makes magnitude one of the most worthwhile things to assess in grades 3–6 (Geary, 2011; Schneider et al., 2018).

Number-line and comparison items reveal magnitude reasoning directly: *Place 472 on a number line from 0 to 1,000. Which is greater, 509 or 590, and how do you know? About how much is $6,000 - 2,950$?* The most informative version asks for justification. A student who explains the comparison by noting that, after the equal hundreds, 590 has nine tens to 509’s zero tens is reasoning about quantity; one who simply says “590 has a bigger 9” may be comparing digits. Estimation prompts — *closer to 3,000 or 4,000?* — surface whether students reason about the total before computing.

Key Idea • Magnitude understanding — knowing where a number falls, not just how to write it — is among the strongest predictors of later math achievement; number-line placement, comparison-with-justification, and estimation items assess it efficiently.

The Multiplicative Structure of the Places

In grades 4–5, students grasp that the places form a multiplicative structure: **each place is worth ten times the place**

to its right and one-tenth of the place to its left. This is the property that distinguishes a true base-ten understanding from a memorized list of place names (Ross, 1989). It is also what makes multiplication and division by powers of ten meaningful rather than a matter of “adding zeros,” and it is the bridge into decimals, where the same times-ten and one-tenth relationship continues to the right of the ones place.

Because this relationship is often left implicit, it is frequently the missing piece when students struggle with decimals or with scaling. A student who understands that moving a digit one place to the left multiplies its value by ten has a structural account of why 0.1 is ten hundredths and why 30 is ten times 3. Making this relationship an explicit object of instruction — and of assessment — is among the most useful moves a teacher can make in these grades (National Research Council, 2001).

Items that probe the multiplicative relationship are especially revealing: *What happens to the value of a digit when it moves one place to the left? How many times greater is the 7 in 7,000 than the 7 in 70? How many tenths are in 5? How many hundredths are in 0.5?* A student who answers “ten times” and can connect it to both whole numbers and decimals holds the structural understanding; one who treats each place as unrelated will often fall back on “adding a zero” and falter when decimals make that rule fail.

Key Idea • Each place is worth ten times the one to its right — the multiplicative structure that turns a list of place names into base-ten understanding; assess it with “how many times greater” and “what happens when a digit moves places”.

Equivalence and Flexible Renaming

A supporting understanding ties units and magnitude together: the recognition that **the same quantity can be represented in different but equivalent ways**. The number 47 is 4 tens and 7 ones, but it is equally 3 tens and 17 ones, $40 + 7$, or $50 - 3$. These equivalences are not arithmetic trivia; they are what make flexible computation possible. A student who can rename 304 as 2 hundreds, 10 tens, and 4 ones is prepared to subtract across zeros with understanding rather than by rule (Hiebert & Carpenter, 1992).

This is also where the equals sign should be treated as a statement of balance rather than a signal to compute. Place value gives students a natural context for seeing equality as equivalence: different decompositions of the same number are genuinely equal. Students who hold this relational view of equality are better positioned for the structural reasoning that algebra requires (Skemp, 1976).

Renaming and equivalence items reveal flexibility: *Show 304 in two different ways. Explain why 6 tens and 14 ones equals 74. Fill in the blank: $3,000 + \underline{\hspace{1cm}} + 60 + 2 = 4,762$.* A student who can produce multiple correct decompositions, and justify why they are equal, demonstrates the flexible unit reasoning that procedural fluency alone does not guarantee.

Key Idea • The same quantity can be named in many equivalent ways, and flexible renaming is what makes regrouping and mental computation make sense; assess it by asking students to represent a number in more than one way and justify the equivalence.

Reading the Errors: What Common Misconceptions Reveal

Place-value errors are rarely careless; they usually point to a specific gap in unit or magnitude understanding, which is what makes them so useful formatively. The most common pattern is treating multi-digit numbers as digits placed side by side — reading 36 as “3 and 6” rather than 3 units of ten and 6 units of one. Such a student can often name the tens place while still not understanding that the 3 represents 30 (Ross, 1989).

A second pattern is treating regrouping as a rule rather than a unit relationship. Students who “borrow” or “carry” by procedure often cannot explain that one ten is being partitioned into ten ones. The language matters: describing the action as composing (iterating) and decomposing (partitioning) units, rather than borrowing and returning something, keeps the focus on the mathematics (Skemp, 1976). A third pattern concerns zero as a placeholder — a student who does not understand that the 0 in 406 marks the absence of tens may write four hundred six as 4006 or 46.

A fourth pattern, increasingly important with decimals in grades 5–6, is “longer means larger” reasoning — judging 0.38 to be greater than 0.4 because it shows more digits. This error is a direct signal that magnitude understanding has not yet extended to decimals, and it is precisely what a well-chosen comparison item can surface. In each case the error is diagnostic: it tells the teacher which understanding to rebuild, not merely that an answer was wrong.

Key Idea • Place-value errors are diagnostic, not careless — digits-side-by-side, regrouping-as-rule, zero-as-placeholder, and longer-means-larger each point to a specific gap in unit or magnitude understanding that targeted instruction can address.

Conclusion

Place value becomes powerful when students understand that every number is built from units — and that those units can be composed, decomposed, compared, iterated, partitioned, and represented in multiple equal ways. For students in grades 3–6, this is not review of a primary-grade topic but the foundation on which multi-digit operations, decimals, fractions, and early algebra are built.

When students struggle, the issue is usually a developmental gap in unitizing or magnitude rather than carelessness, and the research is clear that these are among the understandings that predict long-term mathematical success. Assessment that targets units, magnitude, the multiplicative structure of the places, and flexible equivalence — and that listens for reasoning rather than recording answers — gives teachers an early, actionable read on where students stand and what they need next.

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Social Media Post

Your students can name the places in 5,026. So why does place value keep tripping them up in grades 3–6?

Because naming positions isn't the same as understanding the units a digit represents — or how large the number actually is.

Our latest DMTI Insight on Place Value looks at what to assess in grades 3–6, and why these understandings predict long-term math success:

- **Units over positions.** Knowing how many complete tens are in 340 tells you more than naming the tens place ever will.
- **Magnitude is a predictor.** Number-line placement is one of the most robust predictors of later math achievement.
- **The places are multiplicative.** Each place is ten times the one to its right — the structure that bridges whole

numbers and decimals.

- **Errors are diagnostic.** “0.38 is bigger than 0.4” tells you exactly which understanding to rebuild.

This isn’t about reviewing a primary-grade topic. It’s about assessing the reasoning that carries students from whole numbers through fractions, decimals, and algebra.

Read the full research overview at <https://dmtinstitute.com/dmt-insights>

Which place-value understanding do your grade 3–6 students find hardest — unitizing, or magnitude? Share your thoughts at contact@dmtinstitute.com.



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