



Developing Mathematical Thinking:
Rethinking the Teaching of Area and Perimeter

DMTI INSIGHTS BY
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RETHINKING THE TEACHING OF AREA AND PERIMETER

(DMT INSIGHTS)

Introduction

Why do the same area and perimeter errors show up year after year, even when these topics are “covered” multiple times?

Students’ most common misconceptions about area and perimeter are not random; they are the predictable result of typical instructional sequences that blur the fundamental dimensional nature of these concepts. When instruction treats area and perimeter as a pair of similar formulas attached to the same diagrams, rather than as distinct measurement quantities, students build fragile, formula-driven schemas that fail outside of routine problems (Battista, 2004; Lehrer & Wilson, 2011). The tools students use often deepen the confusion, as the same square tile may be used ambiguously for both perimeter and area, obscuring whether the edge (1D) or the face (2D) constitutes the unit (Lehrer, Jenkins, & Osana, 1998).

The Core Problem—Obscured Dimensionality and Cognitive Load

How do dimension, units, and cognitive load interact to make area and perimeter harder than they appear?

The central instructional problem is a failure to make explicit the dimensional nature of measurement quantities. Perimeter is a one-dimensional (1D) quantity measuring length around a boundary, while area is a two-dimensional (2D) quantity measuring surface coverage (Clements & Sarama, 2014). Teaching these concepts simultaneously, emphasizing formulas prematurely, or using ambiguous tools, obscures this distinction and leads to systematic misconceptions. From a cognitive psychology perspective, this approach imposes high extraneous cognitive load: students must juggle symbols, visual interpretation, and procedural recall without a clear mental model of what is being measured, overwhelming working memory and hindering schema construction (Sweller et al., 2011; Mayer,

Predictable Misconceptions and Their Instructional Roots

What are the most common errors, and how does current instruction set them up?

Research and classroom evidence reveal a consistent pattern of errors (Battista, 2004):

- **Formula Swapping:** Using length $\times$ width for perimeter or adding sides for area.

- **Area-Perimeter Conflation:** Believing larger perimeter means larger area, or that shapes with equal area must have equal perimeter.
- **Grid-Counting Errors:** Counting boundary squares for area, interior squares for perimeter, or grid intersections instead of units.
- **Lack of Transfer:** Inability to reason about non-rectangular or composite shapes without a procedural cue.

These misconceptions arise predictably from common instructional practices:

1. **Teaching Area and Perimeter Together:** Presenting them in the same unit frames them as procedural variations of the same task, rather than distinct concepts (Battista, 2004).
2. **Starting with Formulas:** Introducing $A = l \times w$ and $P = 2(l + w)$ before establishing the concepts of unit iteration reduces measurement to abstract symbol manipulation (Lehrer et al., 1998).
3. **Using Dimensionally Ambiguous Tools:** Employing the same tool (e.g., square tiles) for both perimeter and area without clarifying the shift from a 1D edge unit to a 2D surface unit sends mixed signals (Clements & Sarama, 2014).

Misconceptions as Diagnostic Feedback

What are students' errors telling us about our teaching?

Student errors are not just mistakes—they are clues about how instruction shaped students' thinking. Area-perimeter conflation signals a lack of experience with contrast tasks in which one quantity varies while the other is held constant (Battista, 2004). Formula swapping indicates that procedures were memorized without connection to the underlying unit structure. Grid-counting mistakes reveal that students were not explicitly taught “what counts as one unit” for each dimension. Failure with irregular shapes indicates an overreliance on formula spotting rather than reasoning through decomposition and unit iteration. These patterns collectively indicate that instruction blurred dimensional distinctions, introduced symbolic shortcuts too early, and used tools that did not consistently embody the intended attribute (Sweller et al., 2011).

A Dimensional Framework for Measurement (0D–1D–2D)

How can a dimensional storyline organize student thinking?

A coherent dimensional progression provides a powerful conceptual framework. Learning trajectories research supports building understanding from foundational 1D concepts to more complex 2D and 3D ones (Clements & Sarama, 2014; Lehrer & Wilson, 2011):

- **1D (Length/Perimeter):** Iterating linear units along a path.
- **2D (Area):** Structuring space into an array of square units and coordinating them multiplicatively. This progression represents a qualitative shift in reasoning. Explicitly discussing “Are we measuring the

boundary (1D) or the surface (2D)?” helps students categorize quantities and select appropriate strategies.

Aligning Tools and Units with Dimensions

Which tools best highlight 1D and 2D measurement, and how should we use them?

Tools must be chosen and used to make dimensionality perceptually obvious (Battista, 2004; Clements & Sarama, 2014):

Dimension	Quantity	Ideal Tools & Units	Purpose
1D	Perimeter	String, ruler, tape; linear units	To embody iteration of length around a boundary.
2D	Area	Square tiles, grid paper; square units	To make surface coverage and array structure visible.

The principle is **intentional alignment**: use tools whose form and function match the dimension of the attribute being measured, and explicitly name that match. Avoid using square tiles to *measure* perimeter, as this conflates the 2D object with the 1D unit. Digital tools should be chosen for their ability to preserve unit visibility and focus attention on structure, not just dynamic manipulation (Mayer, 2020).

An Improved Instructional Pathway

How can we redesign instruction to build lasting understanding?

Effective instruction inverts the common formula-first sequence:

1. **Separate and Establish 1D:** Develop a robust concept of length and perimeter through unit iteration with linear tools.
2. **Build 2D Concept from Units:** Introduce area as *covering* with square tiles. Focus on tiling, counting, and structuring into rows/columns long before naming a formula.
3. **Contrast to Clarify:** Once each concept is stable, use contrast tasks (e.g., fixed perimeter with varying area) to solidify the dimensional distinction (Lehrer & Wilson, 2011).
4. **Generalize with Formulas:** Introduce formulas *only* as efficient records of the unit-iteration processes students already understand.
5. **Apply and Transfer:** Use decomposition and composition tasks with complex figures to reinforce reasoning from units, not shape recognition.

This pathway manages cognitive load by sequencing concepts logically, using supportive tools, and delaying symbolic abstraction until conceptual schemas are formed (Sweller et al., 2011).

Conclusion

The persistent difficulties students experience with area and perimeter are not a mystery, nor are they the result of students “not trying hard enough.” From both mathematics education and cognitive psychology perspectives, these errors are the predictable outcome of instruction that blurs dimensional distinctions, minimizes units, and introduces symbolic shortcuts before conceptual understanding is established.

When perimeter is taught as a one-dimensional quantity that measures length around a boundary, and area is taught as a two-dimensional quantity that measures surface coverage through unit iteration, students are far less likely to confuse the two. When tools clearly embody the attribute being measured and instruction progresses from concrete action to visual structure to symbolic representation, cognitive load is reduced, and understanding becomes durable.

Area and perimeter are not isolated topics; they are gateways to proportional reasoning, algebraic thinking, and spatial sense. Teaching them well requires more than better worksheets or clearer explanations—it requires instructional designs that align mathematical structure with how students learn. When that alignment is present, formulas become meaningful, misconceptions become instructional feedback, and students gain understanding that transfers beyond the page.

References

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Why do students keep confusing area and perimeter—even after we teach it every year?

Because most instruction treats them as *two formulas on the same picture* rather than as *two different kinds of measurement*.

Our latest **DMT Insight** shows:

- Perimeter is a 1-D boundary measure
- Area is a 2-D surface measure
- When units, tools, and dimensions aren't made explicit, misconceptions are guaranteed

This isn't a student problem—it's an instruction problem.

When teachers:

- ✓ Separate 1-D and 2-D thinking
- ✓ Ground formulas in unit iteration
- ✓ Use tools that match the measurement

Students finally understand—and that understanding transfers.

Area and perimeter aren't just topics. They're gateways to algebra and spatial reasoning.

What's one misconception about area or perimeter you see every year?



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