DMT COMPANION

Insight 58 · 2026 · for teachers, coaches, principals, and families

FRACTIONS: THE DEVELOPMENTAL UNDERSTANDINGS BENEATH CONCEPTS AND OPERATIONS

How to Use This

This Companion accompanies the DMT Insight on fractions. Start with the three guiding questions below, on your own or with a team, before you read the Insight or watch the video. Answer them again afterward. The difference between your two answers is where the learning shows. Teachers can use it to plan a unit or to study on their own. Coaches can use it to run a team session. Principals can use it to know what to look for in classrooms. Parents can use it to understand the work and support it at home.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. When a student sees $\frac{3}{4}$, do they see a 3 and a 4, or **one quantity composed of three one-fourth units**?
2. When comparing two fractions, does the student focus on the numerals, or reason about **the unit size and the number of those units**?
3. Can the student locate a fraction on a number line and explain **its magnitude in relation to 0, 1, and other benchmarks**, or only represent it as shaded parts of a shape?

Before and After: Two Whole Numbers, or One Quantity with a Size

Every Insight turns on one shift. Here, the shift is from reading a fraction as two whole numbers to seeing it as one quantity with a size on the number line. In the picture, a student compares one-third and one-fourth. On the left, the student reads two whole numbers and picks the larger digit. On the right, the number line shows that one-third is farther from zero, so one-third is larger.

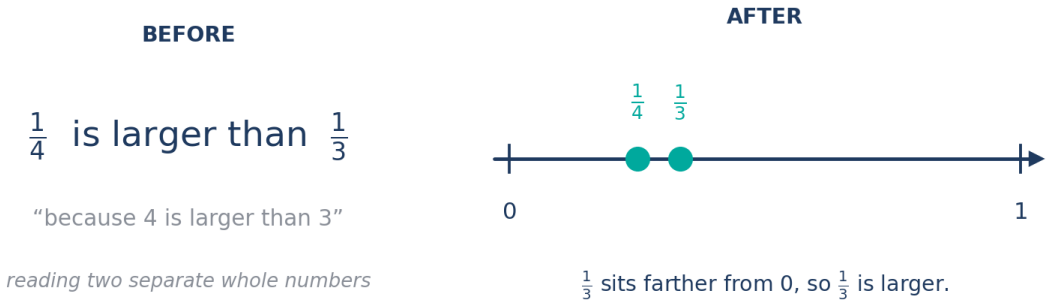


Figure 1. Before: two whole numbers. After: one quantity with a place on the number line.

When a fraction becomes one point with a size, a student can compare it, estimate it, and judge whether an answer is reasonable.

Worked Examples

Build each idea from a visual model first, then attach the words to say. Put the picture before the symbol every time.

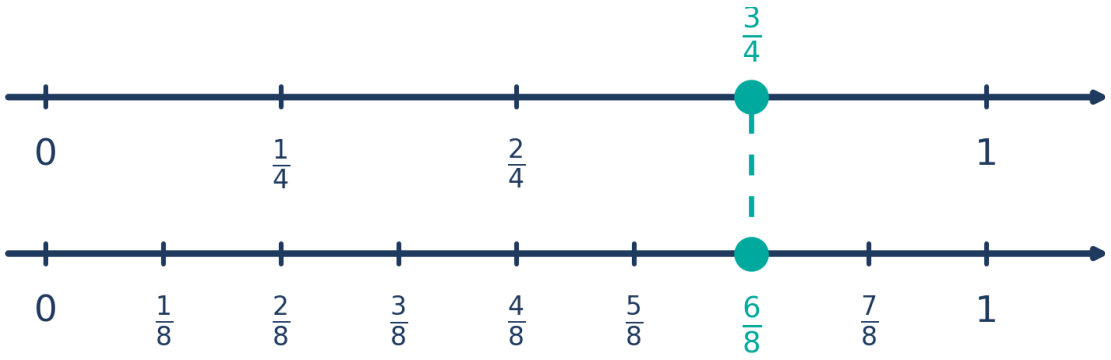
Example 1 · Build a fraction from a unit

Figure 2. Partition one into four equal units, name the unit fraction, then iterate it to three-fourths.

The language to use:

- "What is one here?"
- "We partition one into four equal units. One unit is one-fourth."
- "Count the units: one-fourth, two-fourths, three-fourths."
- "Three-fourths is three copies of one-fourth."

Example 2 · Equivalence as renaming the unit



The same point, named in fourths and in eighths: partition each fourth in two.

Figure 3. Three-fourths and six-eighths land on the same point. Partition each fourth into two.

The language to use:

- "Partition each fourth in half. How many pieces do we have now?"
- "The point did not move. The amount is the same."
- "Three-fourths and six-eighths are the same quantity, named in different units."

The Language Shift

Keep this by the board. The left column is the phrasing that hides the mathematics; the right column is the phrasing that makes it explicit.

Instead of	Say
the top number and the bottom number	the denominator names the unit size; the numerator counts those units
one-eighth is bigger because eight is bigger	more pieces means smaller pieces, so one-eighth is less than one-third
how many pieces are shaded	how far the fraction is from zero
reduce the fraction	rename the same quantity using larger fractional units

Reading Where Students Are

- Unit of 1. “If this bar model is three-fourths, show me one.” Can the student identify and rebuild the unit of one?
- Partitioning. “Partition one into six equal units. Are the units equal, and how do you know?” Does the student see that more equal partitions make smaller units?
- Iterating. “Use one-fifth to build three-fifths. Now use one-fourth to build five-fourths.” Can the student iterate the fractional unit, including past one?
- Magnitude. “Is seven-eighths nearer 0, one-half, or 1? Which is larger, one-third or one-fourth?” Does the student reason about size?
- Equivalence. “Without a rule, why is two-thirds the same as four-sixths?” Does the student rename the unit, or recite a procedure?

Across the Grades (3 to 6)

Grade band	Example	Model	The shift it asks for
Grade 3	Fold a paper strip into 4 equal parts. Where is one-fourth? Where is three-fourths? Place them on a number line from 0 to 1.	Bar model, number line	From “pieces” to a unit you can name, place, and count.
Grades 4-5	Show why three-fourths and six-eighths are the same amount by partitioning each fourth in two. Which is closer to 1, seven-eighths or five-sixths?	Number line, area model	From a rule for equivalence to renaming the same quantity in smaller units.
Grade 6	Find three-fourths of 12. How many one-eighths fit in three-fourths? Compare 3 cups for 2 batches on a double number line.	Bar model, double number line	From “invert and multiply” to reasoning about how many units fit and how quantities scale.

Where to Go Next

For the full research overview and the video, visit mathsuccess.io/dmtinsights.

To access the Math Learning Profile (MLP), Targeted Activities, and more, visit mathsuccess.io.