DMT COMPANION

Insight 56 · 2026 · for teachers, coaches, principals, and families

GEOMETRIC MEASUREMENT: BEFORE THE FORMULAS

How to Use This

This Companion accompanies DMT Insight 56, Geometric Measurement: Before the Formulas. Teachers can read it alone to plan. Coaches can run a session from it. Principals can use it to know what to look for. Families can use it to see what the work at home supports. One move makes it all work, and it takes about a minute: when a student gives you a measurement, ask what the student counted. A correct number can come from counting equal units, reading a mark, or applying a rule. The two students need different next lessons.

> **Key Idea • Ask what the student counted. A correct answer does not indicate which idea produced it.**

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. What unit has the student established, and how is that unit being iterated or composed?
2. Is the student reasoning about the magnitude of the attribute, or reading marks, numerals, or a procedure?
3. How is the student organizing the units and relationships among them, and what remains the same when the figure or measure changes?

Before and After: What the Student Is Counting

Place a pencil beside a ruler so that it begins at 3 and ends at 8, and ask how long it is. A student who has learned the ruler as a row of labeled positions reports 8, because 8 is the numeral at the end of the pencil. A student who has built the unit reports 5, because five equal spaces run from where the pencil starts to where it ends. The unit is the space between the marks, not the mark.

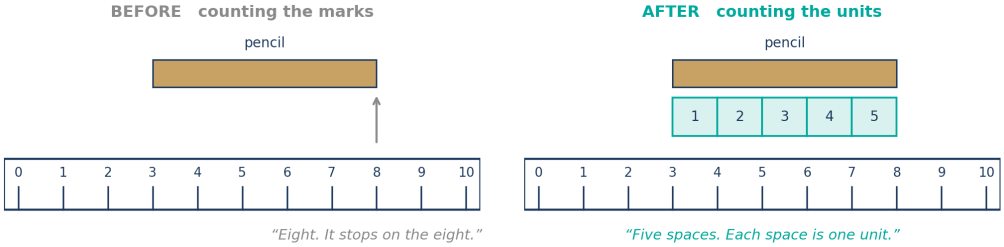

Figure 1. The same pencil and the same ruler, read two ways. Before, the student counts the marks. After, the student counts the equal units the pencil covers.

Every measurement a student reports should be a count of equal units. If the units are not visible in the diagram, they may not be there in the reasoning. This task separates Unitizing from Measurement Magnitude. A student may establish an equal-sized unit and still fail to coordinate the starting point with the accumulated intervals. The unit is not the problem; the ruler is.

> **Key Idea • Build the unit before you hand over the tool. The shifted object exposes the difference in under a minute.**

Worked Examples

Each example gives a model students can reason with and the language that directs attention to the mathematical structure. Show the visual model before or alongside the symbols, and connect the two explicitly.

Example 1 · Spatial Structure: area as rows of square units

Draw a rectangle with no grid inside it and ask students to show the units before anyone computes. A student who draws squares of convenient size around the edges has not built the array. In the Insight, a fourth grader shown only the top row and left column of the squares counts 7; asked to finish drawing them, she fills the region row by row and writes 12. A student fills the rectangle with equal square units, then sees eight in a row and six rows, and has built it. The array shows six rows of eight, and it also shows $(5 \times 8) + (1 \times 8)$, which is the distributive property before it becomes a formula.

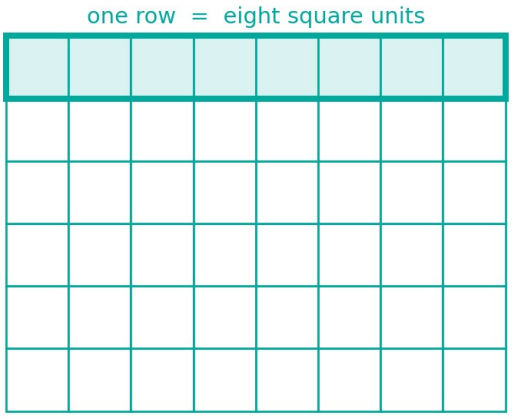

Figure 2. A complete array of equal square units, with one row marked. Six rows of eight compose 48 square units.

- *"Show me one square unit. How many of those fit in one row?"*
- *"How many rows like that one fit in the rectangle?"*
- *"Where is the 8 in your diagram? Where is the 6? What does the 48 count?"*

Example 2 · Relational Thinking: area and perimeter

Area measures the space inside a figure—perimeter measures the length of its boundary. The two measures can change independently. Give students 24 units of fencing and ask them to build every rectangle with a perimeter of 24 units. Then reverse the task: build every rectangle with an area of 24 square units and watch the perimeter change.

Students who expect the two measures to move together will predict otherwise out loud, which is the moment the lesson can use.

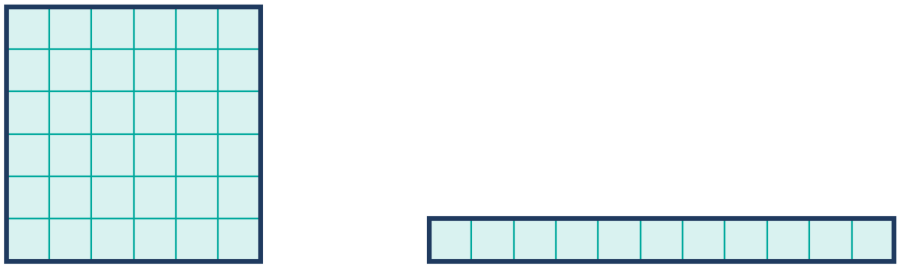

Figure 3. Two rectangles with the same perimeter and very different areas.

- **"Both of these have a perimeter of 24 units. Do they hold the same amount of space?"**
- **"Area measures the space inside the figure. Perimeter measures its boundary. Which one does the question ask for?"**
- **"What stayed the same? What changed?"**

Example 3 · Unitizing and Spatial Structure: cubes, rows, and layers

Build a rectangular prism 5 long, 4 deep, and 3 high from cubes, then take it apart one layer at a time. Ask how many cubes are in one layer before anyone counts the whole prism. Students who count only the cubes they can see will miss the interior, so let them take it apart and find out.

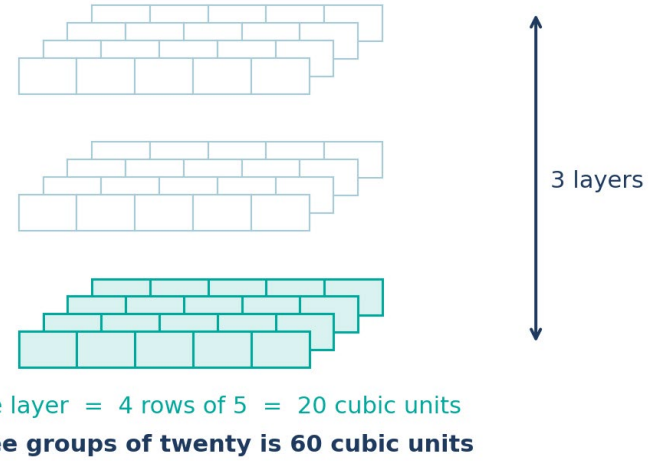

Figure 4. One layer is four rows of five, which composes 20 cubic units. Three layers compose 60.

- *"How many cubes are in one layer? How did you get 20 without counting each one?"*
- *"How many layers are there? So how many cubes altogether?"*
- *"You counted three groups of twenty instead of sixty cubes. Where else do we count that way?"*

Language That Makes the Structure Visible

Keep this by the board. The left column is wording that lets a student answer without the idea. The right column directs attention to the unit and the count.

Instead of	Say
How long is it?	How many of these units fit along it?
It ends on the 8.	How many spaces are there from where it starts?
Length times width.	How many square units in one row? How many rows?
Count the squares.	Show me one square unit, then show me a row of them.
Find the perimeter and the area.	Area measures the space inside—perimeter measures the boundary.
Multiply the three numbers.	How many cubes in one layer? How many layers?
Which one is larger?	Which one holds more square units? How do you know?
Which angle is bigger?	Which represents a greater amount of turn? How do you know?
What happens to the perimeter?	What stayed the same? What changed about area and perimeter?

What You Should See and Hear

- Students establish a unit and iterate it before a ruler or a formula appears.
- Students draw the units in a diagram before they compute, and the diagram shows equal units with no gaps or overlaps.
- Teacher questions name the unit and the count rather than the operation.
- A measurement task appears with the object shifted away from zero at least once.
- Students name a total as a count of units, for example six rows of eight, not only as a product.
- Students are asked what stayed the same and what changed when a figure is rearranged or its dimensions change.

Key Idea • Every question in the right column asks for a unit and a count. That is the shift, said out loud.

Across the Grades

The same structure runs from grade 2 to grade 8. What changes is the attribute measured and the size of the unit students compose.

Band	What students build	Model	What changes
Grades 2–3	Length. Comparing and ordering, iterating a paper strip, and reading a ruler when the object does not start at zero.	Bar model and number line	From counting the marks to counting the equal units

Band	What students build	Model	What changes
Grades 4–5	Area, perimeter, and angle. Covering a region with square units and separating the space inside from the perimeter.	Area model, with the bar model for length	From length times width to rows of square units
Grades 6–8	Volume, surface area, scale, and similar figures. Composing layers and scaling a figure so its shape holds.	Area model and ratio table, with equations last	From a formula to composing units and scaling them

Key Idea • Same structure, larger units. A grade 6 student composing layers is doing what a grade 2 student did with a paper strip.

Reading Where Students Are

The Geometric Measurement Math Learning Profile (MLP) reports four developmental understandings: Unitizing, Measurement Magnitude, Spatial Structure, and Relational Thinking. A pattern of responses points to the understanding underneath the answer, not to a missed item.

A student may establish and iterate an equal-sized unit correctly and still read a ruler as numerals rather than as intervals accumulated from a starting point. That pattern points to Measurement Magnitude, not Unitizing. The unit is not the problem; the ruler is. Start with a measurement that begins at 2, then ask why the answer is not the number it ends on. Disconfirming evidence changes the strategy; more aligned practice does not.

A student who expects area and perimeter to move together has fused the two measures into one idea. That pattern points to Relational Thinking, not to more practice calculating area and perimeter. Start by building every rectangle with an area of 24 square units and watching the perimeter change, then every rectangle with a perimeter of 24.

A student who counts every square rather than using rows and columns, or who recalls a formula without being able to show the units, is showing what the MLP reports separately as Strategy Flexibility. It describes how a student uses the structure rather than naming another measurement understanding.

Key Idea • Read the pattern, not the item. The same wrong answer can come from Unitizing, Measurement Magnitude, Spatial Structure, or Relational Thinking, and each one points to a different next lesson.

Where to Go Next

Read DMT Insight 56, Geometric Measurement: Before the Formulas, and watch the companion video at mathsuccess.io/dmtinsights. Bring the three guiding questions to your next team meeting, answer them again after reading the Insight, and choose one worked example to try this week. For more resources, visit mathsuccess.io and dmtinstitute.com. This issue's newsletter includes a one-page version to share with families and administrators.

Key Idea • Read the Insight, watch the video, and try one worked example this week.