DMT COMPANION

Insight 55 · 2026 · for teachers, coaches, principals, and families

PATTERNS AND EQUALITY: FROM COMPUTING A TOTAL TO REASONING ABOUT A RELATIONSHIP

How to Use This

This Companion accompanies DMT Insight 55 for educators. Work the three guiding questions before the Insight, and again afterward. The move that makes it work is small. Put an operation on both sides of the equal sign, ask why rather than what, and listen to the explanation instead of scoring the answer.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. When your students see 8 + 4 = ___ + 5, how many write 12 or 17, and what does that tell you that a page of correct addition does not?
2. Which of your students can give the tenth term of 3, 6, 9, 12 without listing the terms in between?
3. When a student says commutative, can that student explain why the order cannot change the total?

Before and After: What an Answer Shows and What Reasoning Shows

Two students can write the same number in the blank but reason very differently. One computes the left side and records the result. The other reads the equal sign as a statement that both sides name one quantity, and adjusts. Ordinary arithmetic cannot tell the two apart, because it never asks.

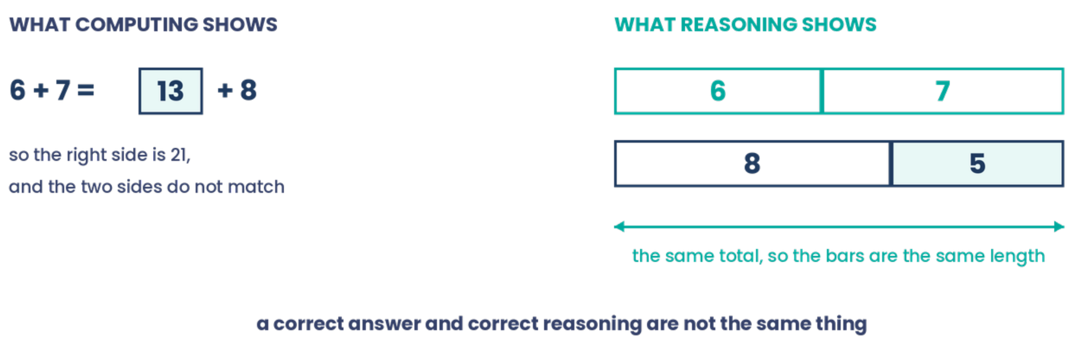

Figure 1. The same item answered two ways. Computing the left side and writing it in the blank leaves the two sides unequal. Reading the equal sign as "the same as" gives two bars of the same length, because both expressions name the same quantity.

The takeaway: a correct answer indicates that a student computed. An explanation of why both sides name the same quantity indicates that a student reasoned.

Worked Examples

Each example pairs a model students should draw with the language that keeps the reasoning with the student. Every task is built so that computing left to right does not settle it. Each is tagged to one of the three MLP understandings, and each understanding carries its own question. Equality as a Relationship: Do both sides name the same quantity? Patterns: What relationship generates the pattern? Algebraic Reasoning: What changed? By how much? What happened to the related quantity? What stayed the same? Algebraic Reasoning here means reasoning from a known relationship or property, not formal algebra.

Example 1 · Equality as a Relationship

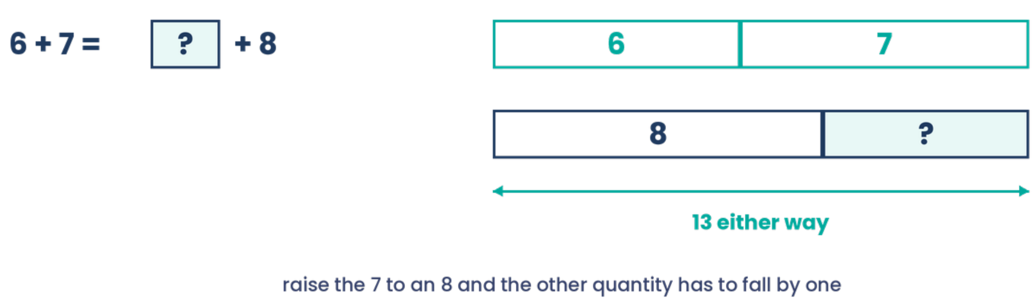

Figure 2. Two bar models of the same length. The unknown is whatever makes the second bar name the same quantity as the first.

A student who writes 13 in the blank has read the equal sign as an instruction. Drawing the two bars shows that both sides name the same quantity before any arithmetic happens, and the missing quantity follows from the length rather than from a sum.

- "What has to be true about the two bars, and why?"
- "You raised the 7 to an 8. What has to happen to the other quantity?"
- "Is 6 + 7 = 13 + 8 a true sentence? How do you know?"

Example 2 · Patterns · Generalizing a pattern

term number	1	2	3	4	n
term	3	6	9	12	3 × n

each term is three times its term number

read across the table for the rule, not down it for the next number

Figure 3. The pattern in a two-column table. The rule lives in the relationship between the columns, not in the step from one row to the next.

A student who reads down the table extends the pattern. A student who reads across describes the rule, and the rule later becomes a variable expression. Ask for a term far enough away that listing is unattractive. The progression runs from noticing what changes, to describing the relationship, to using it to find a distant term, to generalizing it.

- "What is the tenth term, and how do you know without listing?"
- "Tell me the rule without telling me the next number."
- "Will the rule always work? How can you be sure?"

Example 3 · Algebraic Reasoning · The properties as structure

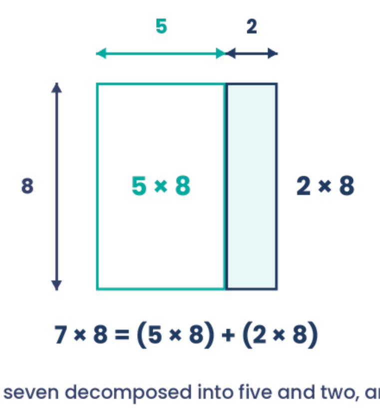

Figure 4. Seven decomposed into five and two. The two rectangles cover the same area as the one rectangle, so the total does not change.

A student who supplies the word distributive and then multiplies both sides has the label. A student who partitions the side of seven and explains why the area stays the same has the structure. Block the computation and the difference appears.

- "Without finding both products, is 6 × 4 = 3 × 8 true? Explain."
- "How does knowing 5 × 8 help you find 7 × 8?"
- "What did you compose? What did you decompose? Did the total change?"

Example 4 · Algebraic Reasoning · Compensation

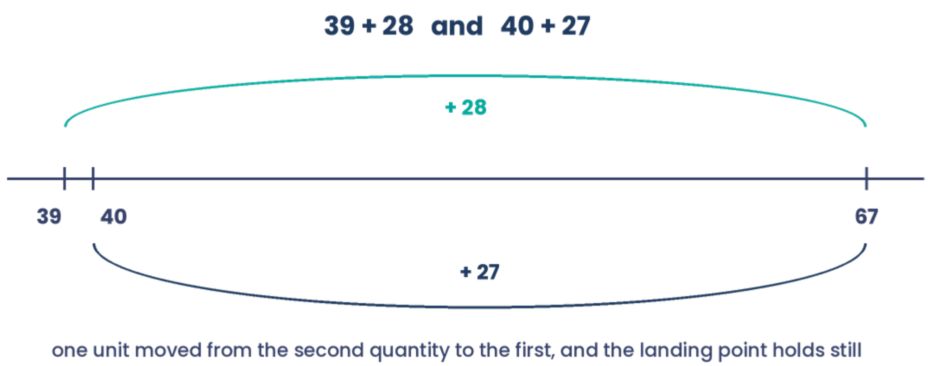

Figure 5. The same total reached two ways. One unit moves from the second quantity to the first, and the landing point does not move.

Compensation puts equality inside arithmetic students already do. The characteristic error is one-sided: a student raises 49 to 50 and leaves the 26 alone, changing the total without noticing. The number line shows what must stay the same. Decompose one unit from the 28, compose it with the 39, and preserve the total.

- "Is 37 + 48 = 40 + 45 true? Explain without adding."
- "If I change 49 to 50, what must I do to 26 so the sum stays the same?"
- "If I double one factor, what must happen to the other?"

Example 5 · Across all three · The unknown and the variable

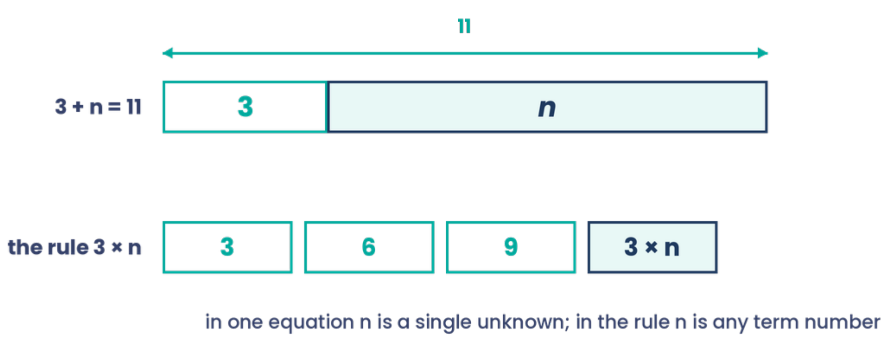

Figure 6. The same letter in two roles. In an equation, it names one unknown quantity. In a rule, it stands for any term number.

A student who treats a letter as a label or a blank will later write expressions that cannot be evaluated or transformed. Build the relationship as a bar model, then as a table, and only then as a symbol.

- "If n + 5 = 12, what is n, and what does n mean here?"
- "A pattern's rule is 4 × n. What does n stand for?"
- "Are n + n and 2 × n the same? How do you know?"

The Language Shift

Wording can either settle the question for the student or hand it back. Keep this by the board.

Instead of	Say
What is the answer?	Is the sentence true? How do you know?
Fill in the blank.	What has to be true about both sides?
What comes next?	What is the tenth term, and how do you know?
Use the commutative property.	What did you compose or decompose? Did the total change?
Make it an easier problem.	You changed one quantity. What has to happen to the other?
Solve for n.	What does n stand for here?

What to look for in a classroom

- Equations written with an operation on both sides, not only in the form a + b = c.
- Students deciding whether a sentence is true before they compute anything.
- Explanations that name what changed and what stayed the same.
- Distant terms predicted from a rule rather than reached by listing.
- Properties described as composing and decomposing before they are named.
- Wrong answers kept, because each one names the understanding to rebuild.

Across the Grades

Grade	What students build	Model	What changes in their thinking
3	Relational equality, and using a pattern's change to find another term.	Bar models of equal length, and a two-column table.	The equal sign stops being a signal to compute.
4	An operation on both sides, and a distant term found by a general rule.	Bar models, number lines, and tables.	The general relationship replaces the next number as the goal.
5	Equalities justified without computing, and compensation in multiplication.	Area models and number lines.	A rewrite is understood to preserve quantity.
6	A letter for a varying quantity, and two expressions compared.	Bar models with an unknown, and tables that become expressions.	The symbol names a quantity, and structure comes before the answer.

Where to Go Next

Read the full research overview on the DMT Insights page at mathsuccess.io/dmtinsights. The Math Learning Profile for Patterns and Equality reports where each student stands across three connected developmental understandings: Equality as a Relationship, Patterns, and Algebraic Reasoning.