

DMT COMPANION

Insight 54 · 2026 · for teachers, coaches, principals, and families



MULTIPLICATION AND DIVISION: THE UNDERSTANDINGS THAT BUILD MULTIPLICATIVE THINKING

How to Use This

This Companion accompanies the DMT Insight on multiplication and division, for teachers, coaches, principals, and families. Work through the guiding questions before the Insight or the video, and again afterward. One useful move is to ask students to model a situation and name the unit before relying only on the fact or the algorithm. Ask: What is the unit being counted? How many of those units? How large is the total?

Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

- 1. Unit & Iterating.** What is the unit in a problem like 4×6 , and what does it mean to iterate it? Can students name six as the unit, taken four times, rather than counting by ones?
- 2. Coordinating Two Quantities.** In $4 \times 6 = 24$, can students hold the number of units, the size of each unit, and the total together, and find any one from the other two (for example, $24 \div 6$)?
- 3. Recognizing Structure.** Can students see the same relationship across representations and break a factor apart to make it easier, renaming 7×16 as $7 \times 10 + 7 \times 6$?
- 4. Comparison & Scaling.** Can students say how many times as many, not only how many more (for example, three times as many rather than eight more)?

Before and After: A Unit, Not a Count

Many multiplication errors reveal a similar habit: students attend to the count of ones without treating a group as a single unit. A before-and-after makes the shift visible, from adding a number again and again to iterating one unit a set number of times.

Before: counting the ones

$$6 + 6 + 6 + 6$$

"6, 12, 18, 24"

attends to the running total, not the unit

After: iterating the unit

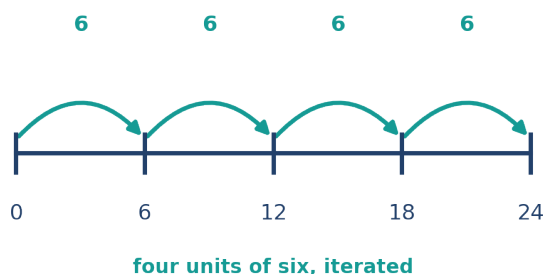


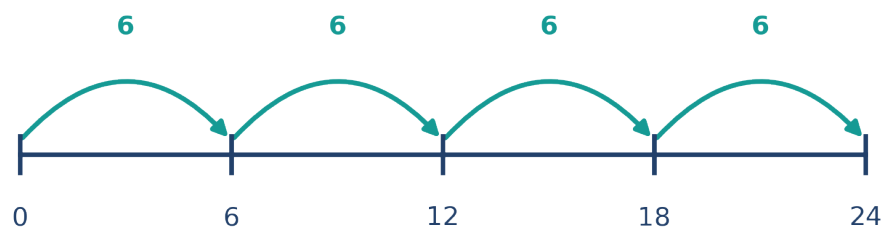
Figure 1. Before, a student counts by ones and reads 4×6 as a running total. After, the student sees four units of six iterated on a number line.

The takeaway: counting the ones tells you only that a student reached the total. Iterating the unit shows that the student understood how the total was built.

Worked Examples

Each example pairs the model students should reason with and the language that directs attention to the unit, the number of units, and the total.

Example 1 • Four sixes as a unit iterated



$$4 \times 6 = \text{four sixes, the unit 6 iterated four times} = 24$$

Figure 2. Four sixes: the unit six, iterated four times, reaches twenty-four.

Unit and iterating is where counting by ones is easiest to mistake for multiplying. On the number line, each equal jump is one unit, and the total is built by iterating that unit, not by counting the ones inside it.

- “Six is the unit. How many sixes do we have, and how far do they reach?”
- “Four jumps of six land on twenty-four, so 4×6 is four sixes.”
- “Counting 6, 12, 18, 24 gives the total, but the unit is the six we keep iterating.”

Example 2 • From 4×6 to $24 \div 6$

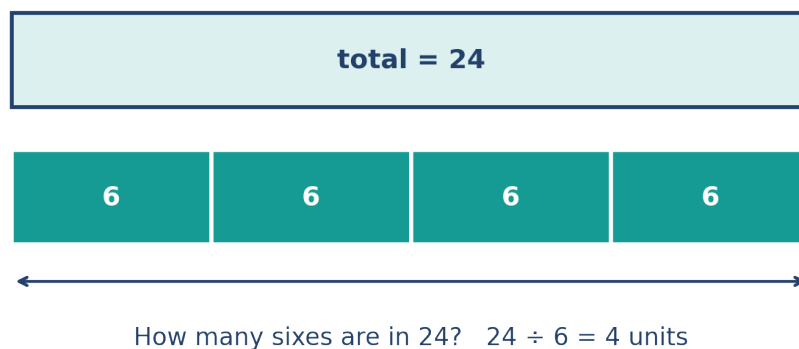


Figure 3. The total twenty-four, seen as four units of six. Division asks how many units make the total.

Coordinating means holding three things at once: the number of units, the size of each unit, and the total. Multiplication and its related division are the same relationship read in different directions.

- “We know the total is 24 and each unit is 6. How many units is that?”

- “ $24 \div 6$ asks for the missing number of units, not a brand-new problem.”
- “Change the total or the unit, and the number of units changes with it.”

Example 3 • Breaking a factor apart: 7×16

$$7 \times 16 = 70 + 42 = 112$$

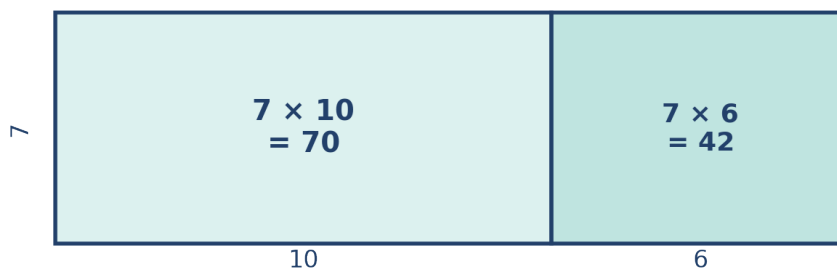


Figure 4. Seven sixteens as seven tens and seven sixes: 70 and 42 recompose to 112.

Recognizing structure lets students decompose a factor, use the distributive property, and recompose the partial products. The same relationship holds whether it is written as an equation or drawn as an area.

- “Break 16 into 10 and 6. What is 7×10 , and what is 7×6 ?”
- “Seventy and forty-two put back together make 112.”
- “The two partial products are parts of one rectangle, not two separate problems.”

Example 4 • Times as many, not more

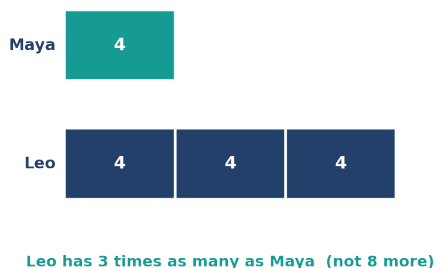


Figure 5. Maya has four; Leo has three times as many, twelve. The comparison is multiplicative, not additive.

Comparison and scaling asks how many times as large, not how many more. A student who says Leo has eight more is comparing additively; the multiplicative comparison is three times as many.

- “Maya has four. Leo has three times as many. How many is that?”
- “Three times as many means three copies of Maya’s four, which is twelve.”
- “Eight more and three times as many describe different relationships. Which does the problem ask for?”

The Language Shift

Wording can hide the mathematics or point at it. Keep this by the board.

Instead of	Say	Mathematical idea
Times means add it again and again.	How many groups, and how big is each group?	Unit & Iterating
Just memorize the facts.	What is the unit, and how many of them make the total?	Coordinating Two Quantities
Multiply the big numbers.	Break a factor apart and add the parts.	Recognizing Structure
How many more?	How many times as many?	Comparison & Scaling
Multiplication makes it bigger.	Will the total grow, and by what factor?	Comparison & Scaling / Estimation

What to Look For in a Classroom

- “Students naming the unit, so 4×6 is four sixes rather than a count to twenty-four.”
- “Students moving between 4×6 and $24 \div 6$ as one relationship, not two problems.”
- “A factor broken apart, 7×16 read as 7×10 and 7×6 , then recomposed.”
- “Comparison stated as times as many rather than how many more.”
- “An estimate of the size of a product before any calculating.”
- “Student drawings kept as evidence, because a model shows the reasoning a fact cannot.”

Developmental Understandings

These develop together rather than in a fixed order, and a diagnostic can look for each.

Developmental understanding	What it looks like when it is emerging
Unit & Iterating	Treats a group as a single unit while knowing what composes it; composes, decomposes, and iterates equal-sized units rather than counting by ones.
Coordinating Two Quantities	Holds the number of units, the unit size, and the total together, and determines any one from the other two, including the related division.
Recognizing Structure	Sees the same multiplicative relationship across representations; decomposes a factor and uses the distributive property to build and recompose partial products.
Comparison & Scaling	Reasons about how many times as many or as large, and distinguishes multiplicative from additive comparison.

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. This issue's newsletter includes a one-page version for families and staff meetings. For more classroom materials, visit mathsuccess.io.