

DMT COMPANION

Insight 53 · 2026 · for teachers, coaches, principals, and families



BEYOND THE ALGORITHM: BUILDING THE REASONING BEHIND ADDITION AND SUBTRACTION

How to Use This

This Companion accompanies DMT Insight 53 for educators. It is organized around the four connected constructs of the Addition & Subtraction Math Learning Profile (MLP): Quantity & Magnitude, Composing & Renaming Units, Structure & Properties, and Estimation. Work the three guiding questions before the Insight or the video, and again afterward. The move that makes it work is small: name the reasoning the lesson is meant to build, find the part the algorithm is doing, and give that part back to the student.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. Which construct is the student using here: Quantity & Magnitude, Composing & Renaming Units, Structure & Properties, or Estimation?
2. What does the student reason out about the quantities, units, structure, or approximate result, and what does the procedure decide for them?
3. Which construct should instruction target next, based on the student's response pattern?

Before and After: What the Algorithm Supplies and What the Student Builds

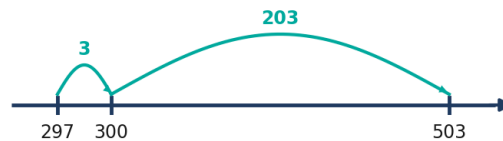
The work a student does is the work a student learns. When the standard algorithm supplies the steps, every decision about magnitude, units, structure, and the size of the result has already been compressed into the procedure. The same computation, reasoned by the student on a number line or a bar model, asks for the thinking the lesson exists to build.

BEFORE · the algorithm lines up the digits

$$\begin{array}{r} 503 \\ - 297 \\ \hline ? \end{array}$$

Every decision about magnitude and units is already inside the steps.

AFTER · the student reasons about distance



The student decides how far it is to the next hundred and how far remains: $3 + 203 = 206$.

Figure 1. The same subtraction twice: the digits the algorithm lines up, beside the distance the student reasons about on a number line.

The takeaway: a completed algorithm shows that a student can run the steps. A drawn, labeled, and explained model shows what the student understood about the quantities and how they relate.

Worked Examples

There is one example for each construct. Each example pairs the model students should build with the language that keeps the reasoning with the student rather than with the algorithm.

Example 1 · Quantity & Magnitude

503 – 297 is the distance from 297 to 503

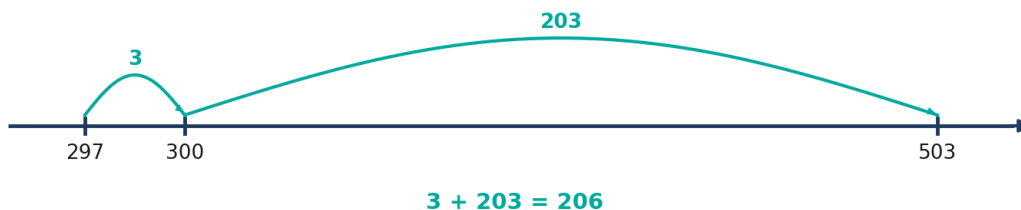


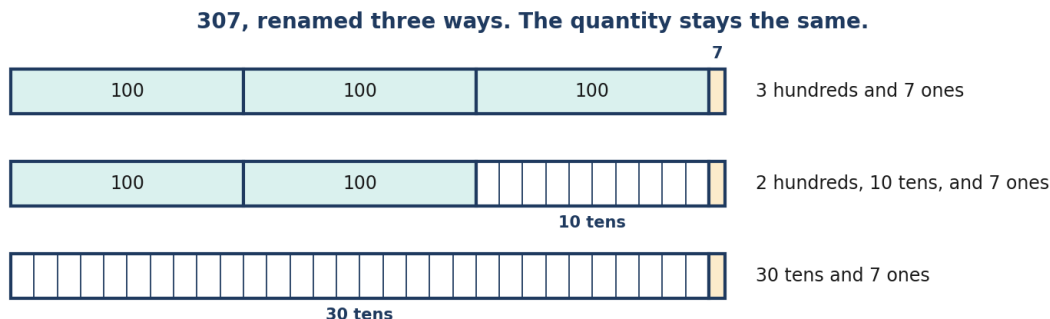
Figure 2. $503 - 297$ on an open number line. Counting up from 297 to 300, then to 503, shows the distance as $3 + 203 = 206$.

The standard algorithm gives $503 - 297$ in a few steps, and a student can run them without noticing that 314 is far too large. Locating 297 and 503 on an open number line takes longer, and the time is where the mathematics is. To draw the jumps, a student decides how far it is to the next hundred and how far remains, a judgment about magnitude that lining up the digits never asks for. A student who reads subtraction as the distance between two quantities counts up from 297 and finds 206, so the answer stays tied to how far apart the quantities are.

- “Where are 297 and 503 located? About how far apart are they? How did you decide?”

- “How far is it from 297 to the next hundred, and how far remains?”
- “What would change on your number line if the problem were $503 - 197$?”

Example 2 • Composing & Renaming Units



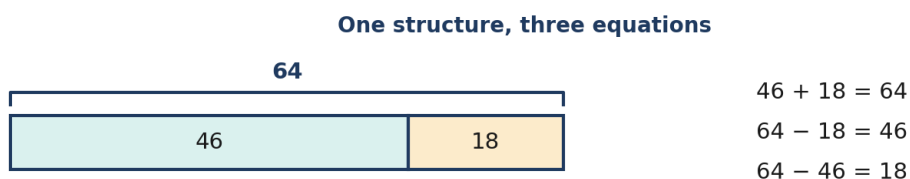
Regrouping renames the units. It does not change the quantity.

Figure 3. 307 renamed in equivalent forms: 3 hundreds and 7 ones; 2 hundreds, 10 tens, and 7 ones; and 30 tens and 7 ones. The quantity stays the same while its unit structure changes.

A student who only follows the borrowing steps regroupes without seeing what changed. A student who names 307 as 2 hundreds, 10 tens, and 7 ones has renamed the units, and on meeting $307 - 8$ can rename one ten as ten ones rather than reaching for a rule. The quantity stays the same while its unit structure changes. Regrouping renames the units; it does not borrow them.

- “Rename 307 another way. Which units did you rename, and how do you know the quantity is the same?”
- “To subtract 8 from 307, what do you need to rename first?”
- “Which renaming of 503 makes $503 - 297$ easiest, and why?”

Example 3 • Structure & Properties



Compensation uses the same structure:

$$46 + 18 \rightarrow 46 + 20 - 2 \rightarrow 66 - 2 = 64$$

add 2 too many, then remove 2

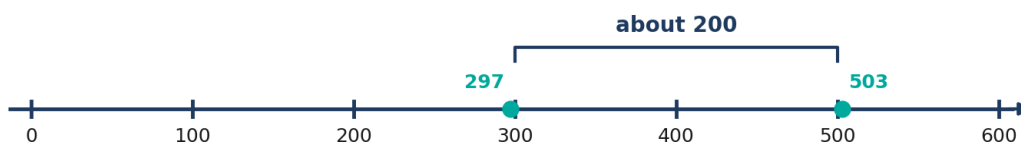
Figure 4. One bar model, three equations: $46 + 18 = 64$, $64 - 18 = 46$, and $64 - 46 = 18$. Compensation uses the same structure.

A student who knows $46 + 18 = 64$ and sees the structure of the bar model also knows $64 - 18 = 46$ and $64 - 46 = 18$, because the same two parts compose the same total. The same structure supports compensation and derived facts: $46 + 18$ can be solved as $46 + 20 - 2$, and $64 - 18$ as $64 - 20 + 2$. The standard algorithm uses the same steps for every problem. A student who reads the structure first chooses a strategy that fits the quantities.

- “If $46 + 18 = 64$, what is $64 - 18$? What is $64 - 46$? How do you know without computing?”
- “How could $46 + 20$ help you find $46 + 18$? What do you need to adjust?”
- “What structure do you notice? Which strategy fits these quantities?”

Example 4 • Estimation

Before computing $503 - 297$: about how large should the result be?



297 is close to 300, and 503 is close to 500.

Could 314 be reasonable? No. The result should be close to 200.

Figure 5. Estimating $503 - 297$ before computing. 297 is close to 300, and 503 is close to 500, so the result should be about 200.

Before computing $503 - 297$, a student can ask how large the result should be. 297 is close to 300, and 503 is close to 500, so the difference should be about 200. A student who has made that estimate knows that 314 is not reasonable, and knows it without computing exactly. Estimation gives the student a way to judge an answer that the procedure alone does not supply.

- “About how large should the result be?”
- “Which benchmarks are useful?”
- “Could 314 be reasonable? Explain without computing exactly.”

The Language Shift

Wording can either hand the thinking to the algorithm or keep it with the student. The rows follow the four constructs. Keep this by the board.

Instead of	Say
Line up the digits.	Where are the quantities located, and how far apart are they?
Carry the one.	You composed a new ten. Which units did you compose?
Borrow across the zero.	Rename one hundred as ten tens. What units do you have now?
Use the same method every time.	What structure do you notice? Which strategy fits these quantities?
Get the exact answer.	About how large should the result be before you compute?
That is the rule.	Why does that work? Show it on your model.

What to look for in a classroom

- Students estimating before they compute and using the estimate to judge the result.
- Quantities located on a number line by hand, spaced by how far apart they are.
- Students naming which units they renamed when they regroup.
- Flawed drawings kept, because a misplaced jump or a mislabeled unit shows the thinking.
- Practice that asks for a model, a strategy, and an explanation rather than a final answer in a box.
- Students choosing a strategy that fits the numbers instead of using one procedure for every problem.

Across the Grades

All four constructs continue in every grade band. The numbers, units, representations, and complexity change.

Band	How the four constructs appear	Model	What changes in their thinking
K–2	Quantities to 100: locating quantities, composing and decomposing tens and ones, using derived facts, and estimating with benchmarks of 5 and 10.	Number lines and part–whole bar models.	A number becomes a quantity with a size and a location, not digits to process.
3–4	Quantities to 1,000 and beyond: distance on a number line, renaming hundreds and tens, compensation and friendly numbers, and estimating with benchmarks of 100 and 1,000.	Open number lines, bar models, and expanded notation.	Regrouping becomes renaming the units while the quantity stays the same.
5–6	Larger whole numbers and decimals: magnitude and distance, renaming with tenths and hundredths, using structure and properties, and estimating to judge results.	Number lines, bar models, and open equations.	The same reasoning extends to new units and prepares students for algebra.

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. The Newsletter for this issue gives a one-page version for families and for staff meetings. For more classroom materials, visit mathsuccess.io.