

DMT COMPANION

Insight 52 · 2026 · for teachers, coaches, principals, and families



SCALING & COMPARING

How to Use This

This Companion accompanies DMT Insight 52 for teachers, coaches, principals, and families. It examines four related ideas in scaling and comparing: scaling, multiplicative comparison, composite units, and rate reasoning. It shows how they connect to the Multiplication & Division and Patterns & Equality Math Learning Profiles (MLPs). Work the three guiding questions before the Insight or the video, and again afterward. Before reaching for a procedure, ask what quantities are being compared, what factor relates them, and what must remain the same as the quantities scale.

Three Guiding Questions

ANSWER THESE BEFORE THE INSIGHT, AND AGAIN AFTER

1. Is the student reasoning with a factor or an additive difference?
2. Can the student keep the quantities coordinated as one relationship is scaled, iterated, or partitioned?
3. What does the response reveal about the student's multiplicative reasoning or understanding of the relationship between the quantities?

Before and After: What the Factor Preserves

A student asked to enlarge a 4-by-6 photograph to a width of 6 will often make the height 8, because 2 was added to the width and 2 seems fair for the height. The arithmetic is sound, and the result is the wrong picture. Scaling both sides by the same factor preserves the shape.

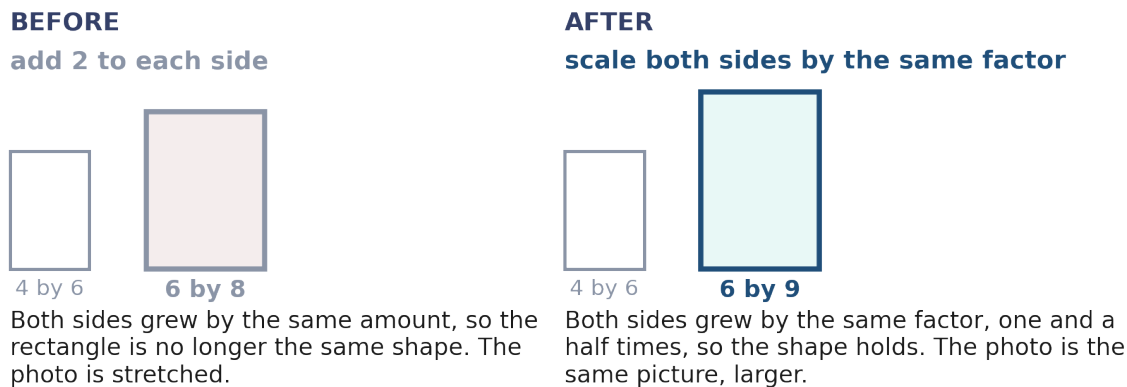


Figure 1. The same enlargement twice: add 2 to each side, then scale both sides by 1.5.

The takeaway: adding the same amount to each quantity changes the relationship between them. Scaling both by the same factor is what leaves the relationship intact.

Worked Examples

The four examples show different ways multiplicative relationships appear. Together they show how students move from acting on one quantity by a factor to coordinating and preserving a relationship between two quantities. Each example pairs the model students should build with the language that returns the comparison to the factor. The throughline is the same in all four: identify the factor, coordinate the quantities, preserve the relationship, and use the relationship to reason.

Example 1 • Scaling

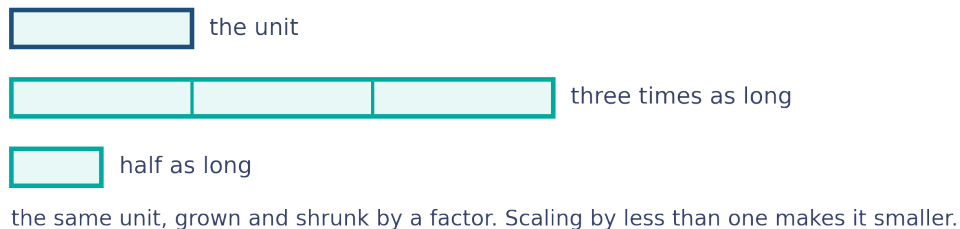


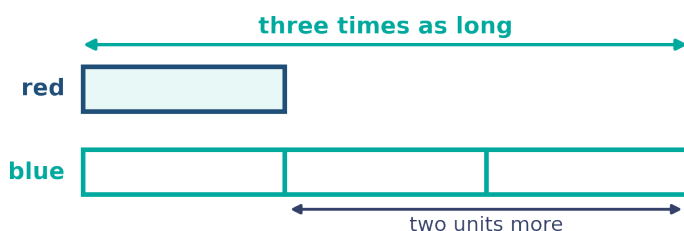
Figure 2. One unit, grown to three times its length and shrunk to half its length.

Scaling acts on a quantity by a factor, not by an amount. In Figure 1, adding 2 to each side of the 4-by-6 photograph gives 6 by 8 and changes the shape. Scaling both sides by 1.5 gives 6 by 9 and keeps the shape. A factor below 1

works the same way: scaling a bar model by one-half makes it half as long. A student who has scaled a bar model by 3 and then by one-half has met the idea that multiplying can make a quantity smaller.

- “Make this bar model three times as long. Now make it half as long. What factor acted on the quantity each time?”
- “What changed when you scaled it? What stayed the same?”
- “You multiplied, and the bar model got shorter. How did that happen?”

Example 2 • Multiplicative Comparison



how many times as much is the multiplicative question. How many more is the additive one.

Figure 3. Two bar models. The blue bar model is three times as long as the red bar model and is also two units longer.

Every comparison can be read in two ways, and each reading answers a different question. How many more asks for a difference. How many times as much asks for a factor. Difference versus factor is the central distinction. Students who read only the difference use additive reasoning in situations that need the factor, and the answer looks reasonable until the numbers change.

- “How many times as long is the blue bar model as the red?”
- “How many more units is it? Which of those two answers did the question ask for?”
- “If the red bar model doubled, what would happen to the factor, and what would happen to the difference?”

Example 3 • Composite Units

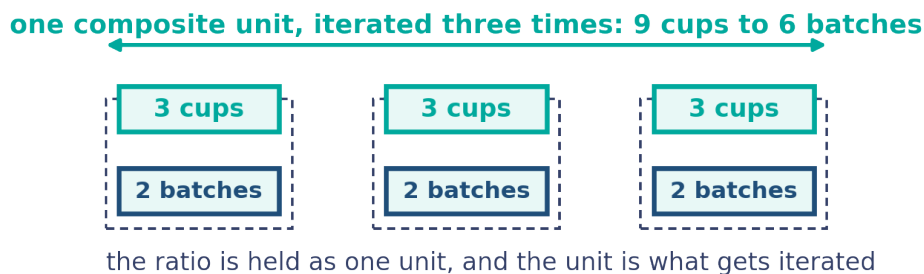


Figure 4. The ratio 3 cups to 2 batches, held as one composite unit and iterated 3 times, gives 9 cups to 6 batches.

A ratio is not two loose numbers. Students compose 3 cups and 2 batches into one composite unit. Iterating that unit gives 6 cups to 4 batches and 9 cups to 6 batches. Partitioning it gives one and one-half cups to 1 batch. Each result is

an equivalent relationship, because the two quantities were iterated or partitioned together. The sequence is unit, compose, iterate, partition, equivalent relationship. It extends the unitizing students use in multiplication: a collection or relationship can be treated as one unit and then iterated or partitioned.

- “What is the one unit that this ratio is built from?”
- “Show me 6 batches. How many times did you iterate the unit?”
- “Partition the unit to show 1 batch. How many cups is that, and how do you know?”

Equivalent Ratios · Preserving the Relationship

		$\times 2$	$\times 3$
scoops	2	4	6
cups	3	6	9
	one unit		

both quantities scale by the same factor, so the mixture tastes the same

Figure 5. A ratio table for a juice mixture: 2 scoops to 3 cups, scaled by two and by three.

Equivalent ratios preserve the relationship because the same factor scales both quantities. Equivalent ratios make the connection to Patterns & Equality especially visible. The quantities change, but the multiplicative relationship remains equivalent. The ratio table makes the factor visible and keeps it attached to what it acts on. A student who scales one row and not the other has changed the mixture, and tasting it is the fastest way to find out.

- “Which column is the unit, and what did you multiply by to reach this one?”
- “What did that factor act on? Show me both rows.”
- “Which of these tastes the same as 2 scoops to 3 cups, and which does not?”

Example 4 · Rate Reasoning

one pound is \$5, and that unit rate builds every other pair



both number lines are scaled together, so the pairing holds all the way down to one

Figure 6. A double number line for 4 pounds at \$20, scaled down to 1 pound at \$5.

A rate coordinates two different units, here pounds and dollars. Scaling both number lines together keeps the pairing intact all the way down to 1 pound, which corresponds to \$5. From that unit rate, a student can build back up to any number of pounds, estimate before computing, and tell whether an answer is reasonable.

- “If 4 pounds cost \$20, what does 1 pound correspond to in dollars?”
- “How did you get from 4 pounds to 1 pound, and what did you do to the dollars?”
- “Build back up from the unit rate. About what would 7 pounds cost? Estimate first, then check.”

The Language Shift

Wording can either settle for a difference or ask for the factor. Keep this by the board.

Instead of	Say
How many more?	How many times as much? Which question does this situation ask?
Add 2 to each side.	What factor did you scale by, and did both sides get the same factor?
A ratio of 3 to 4 is 3 and 4.	Hold the ratio as one unit. What does one of those units give you?
Cross-multiply.	What relationship do the two quantities have? Show how they scale together before formalizing the procedure.
Multiplying makes it bigger.	What happens when you scale by a factor below one? Try half the recipe.

What to look for in a classroom

- Students scale quantities by a factor.
- Students distinguish a multiplicative factor from an additive difference.
- Students treat related quantities as a composite unit that can be iterated or partitioned.
- Students coordinate different units and reason about rates.
- Students explain what changes and what remains invariant as quantities scale.

Across the Grades

These ideas continue in every grade band. Their complexity changes.

Band	What students build	Model	What changes in their thinking
Grade 3	One bag holds 3 apples. Draw 2, 3, and 4 bags, then build a ratio table.	Bar models and ratio tables.	From adding groups to scaling a unit by a factor.
Grades 4–5	A recipe uses 3 cups of flour for 2 batches. Find 6 to 4 and 9 to 6. Which juice tastes the same: 2 scoops to 3 cups or 4 to 6?	Ratio tables and double number lines.	From two loose numbers to one relationship scaled by the same factor.
Grade 6	If 4 pounds cost \$20, what does one pound cost? Enlarge a 4-by-6 rectangle to width 6 without distorting it.	Double number lines and ratio tables.	From a rule to reasoning about rate and about how quantities scale together.

Connecting the Reasoning to the MLPs

Scaling and comparing are not a separate MLP. The reasoning becomes visible across the Multiplication & Division and Patterns & Equality MLPs, and the same response can inform more than one profile.

Student reasoning	What you may see	MLP connection	Next move
Tracks the difference instead of the factor	Adds rather than scales.	Multiplication & Division	Contrast “how many more?” with “how many times as much?”
Cannot hold a ratio together	Changes one quantity but not the other.	Multiplication & Division	Build and iterate a composite unit.
Does not recognize equivalent relationships	Treats 2:3 and 4:6 as unrelated.	Patterns & Equality	Ask what changed and what stayed the same.
Can calculate a rate but cannot explain the relationship	Applies a procedure without coordinating the units.	Both	Model the relationship and connect it to the notation.

Two students can miss the same proportional-reasoning problem for different reasons and need different next moves. One may still be using an additive comparison; another may be applying proportional reasoning where the relationship is not proportional. The answer alone does not tell them apart. The reasoning does.

Where to Go Next

Read the full research overview and watch the accompanying video on the DMT Insights page at mathsuccess.io/dmtinsights. The Newsletter for this issue gives a one-page version for families and for staff meetings. To explore the Multiplication & Division and Patterns & Equality MLPs, Targeted Activities, and more, visit mathsuccess.io.